# Lecture 8: Optical Kerr effect

Learning objectives:

① Intensity-dependent refractive index

② Self-focusing

Recently: $\chi^{(2)}(\omega_3; \omega_1, \omega_2)$ : three wave mixing

Today: $\chi^{(3)}(\omega_4; \omega_1\ \omega_2, \omega_3)$ : Four-wave mixing.

For centrosymmetric media,

$$P^{total} = \underbrace{\epsilon_0 \chi^{(1)} E}_{P_L^{(1)}} + \cancel{\underbrace{\epsilon_0 \chi^{(2)} E^2}_{P_{NL}^{(2)}}} + \underbrace{\epsilon_0 \chi^{(3)} E^3}_{P_L^{(3)}}$$

Input field: $E = E_0 e^{i(kz-\omega t)} + c.c.$

$$\Rightarrow \quad E = E_0^3 e^{3i(kz-\omega t)} + 3E_0|E_0|^2 e^{i(kz-\omega t)} + c.c.$$

$$\text{So } P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} [\underbrace{E_0^3 e^{3i(kz-\omega t)}}_{THG} + \underbrace{3E_0|E_0| e^{i(kz-\omega t)}}_{\text{Optical Kerr effect}} + c.c.]$$

(Selp-phase modulation)

THG $\xrightarrow{\omega,\ \omega,\ \omega} P_N^{(3\omega)}$

Optical Kerr effect $\xrightarrow{\omega,\ \omega,\ -\omega} P_{NL}^{(\omega)}$

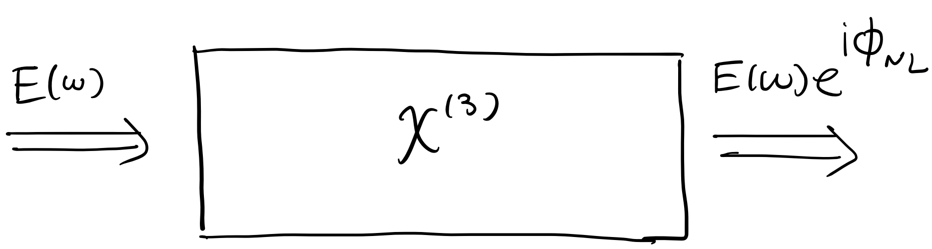
$$\Rightarrow \quad P_{NL}^{(3\omega)} = \epsilon_0 \chi^{(3)} E_0^3 e^{3i(kz-\omega t)} + c.c.$$

$$\boxed{P_{NL}^{\omega} = 3\epsilon_0 \chi^{(3)} E_0 |E_0|^2 e^{i(kz-\omega t)} + c.c.}$$

We will study this today!

## 1. Intensity-dependent refractive index change (Optical Kerr effect)

Problem to study:




A strong beam of light modifies its own propagation!

$$P^{total} = \epsilon_0 \chi^{(1)} E(\omega) + 3\epsilon_0 \chi^{(3)} |E(\omega)|^2 E(\omega) \equiv \underbrace{\epsilon_0 \chi_{eff}}_{\text{effective susceptibility}} E(\omega)$$

$$\Rightarrow \chi_{eff} = \chi^{(1)} + 3\chi^{(3)} |E(\omega)|^2$$

Index: $$n_{eff} = \sqrt{1+\chi_{eff}} = \sqrt{1+\chi^{(1)} + 3\chi^{(3)}|E(\omega)|^2}$$

$$= \sqrt{(1+\chi^{(1)})\left(1+\frac{3\chi^{(3)}|E(\omega)|^2}{1+\chi^{(1)}}\right)}$$

$$\simeq \underbrace{\sqrt{1+\chi^{(1)}}}_{\text{linear ref. index.}} \cdot \left(1+\frac{1}{2}\frac{3\chi^{(3)}|E(\omega)|^2}{1+\chi^{(1)}}\right)$$

$$\Rightarrow \quad n_{eff} = n_0 \left(1+\frac{3}{2}\frac{\chi^{(3)}|E|^2}{n_0^2}\right)$$

$$= n_0 + \underbrace{\frac{3\chi^{(3)}}{2n_0}}_{2\bar{n}_2} |E|^2 \qquad n_{eff} = n_0 + 2\bar{n}_2 |E|^2$$

$$\boxed{\bar{n}_2 = \frac{3\chi^{(3)}}{4n_0}}$$

Usually, we care about input light intensity

$$n = n_0 + n_2 \underbrace{I}_{\text{intensity}}$$

$$= n_0 + 2\bar{n}_2 |E(\omega)|^2$$

$$\Rightarrow 2\bar{n}_2 |E(\omega)|^2 = n_2 I \rightarrow 2 n_0 \epsilon_0 c |E(\omega)|^2$$

$$\Rightarrow \boxed{n_2 = \frac{\bar{n}_2}{n_0 \epsilon_0 c} = \frac{3}{4 n_0^2 \epsilon_0 c} \cdot \chi^{(3)}}$$ Kerr Coefficient

$$n_2 \simeq \frac{283}{n_0^2}\chi^{(3)} \sim 10^{-20}\ m^2/W$$

Example: $CS_2$, $n_2 = 3\times10^{-14}\ cm^2/W$,

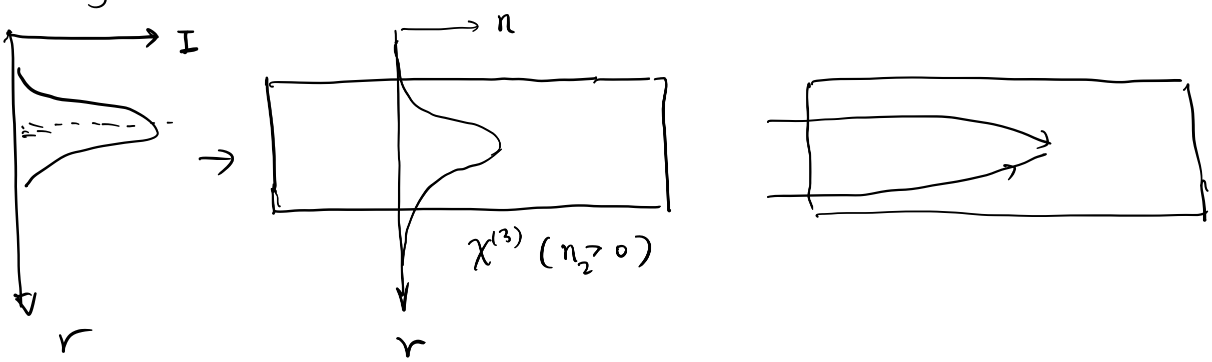
input laser intensity $I = 1\ MW/cm^2$

Index change $= \underline{\underline{3\times10^{-8}}}$. (Seems small, but non-trivial!)

How to enlarge $\Delta n$? Use nanoscale waveguide!

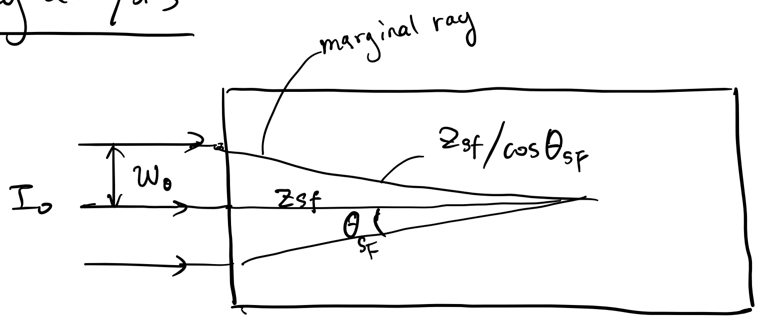
## 2. Self-focusing

Intensity profile




. Laser beam induces a refractive index variation in the $\chi^{(3)}$ medium. larger index at the center. Smaller index at the periphery. Then the $\chi^{(3)}$ medium serves as a lens, causing the beam to focus within the medium.

## Ray analysis




Assumption: ① For marginal rays, $n_0$

② For center rays, $n_0 + n_2 I$

Fermat's principle:

$$(n_0 + n_2 I)\, z_{sf} = n_0 \frac{z_{sf}}{\cos\theta_{sf}} \longrightarrow \text{small angle approximated by } \theta_{sf} = 1 - \frac{\theta_{sf}^2}{2}$$

$$\Rightarrow \theta_{sf} = \sqrt{\frac{2 n_2 I}{n_0}} \quad (\text{Self-focusing angle})$$

$$z_{sf} = \frac{w_0}{\tan\theta_{sf}} \sim \frac{w_0}{\theta_{sf}} = w_0 \sqrt{\frac{n_0}{2 n_2 I}}$$

Let's replace intensity I by power.

Input power: $P = \pi w_0^2 \cdot I$

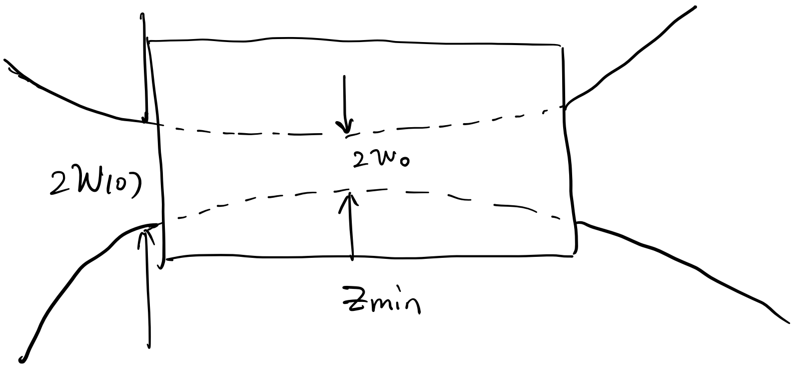
Critical power: $P_{cr} = \dfrac{\lambda_0^2}{8\pi n_0 n_2}$. → critical power for self-trapping

→ higher $n_2$, lower optical power

$$\Rightarrow z_{sf} = \frac{2 n_0 w_0^2}{\lambda_0} \frac{1}{\sqrt{P/P_{cr}}}$$

However, this discussin ignores the diffraction.

Self-focusing effect has to "compete" with beam diffraction




**Modified $z_{SF}$:**

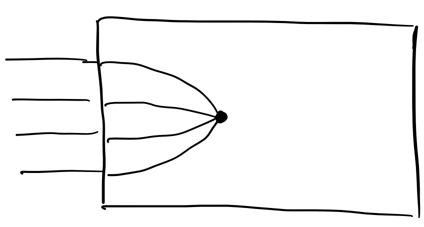
$$z_{SF} = \frac{\pi n_0 w^2(0)}{\lambda_0} \frac{1}{\sqrt{P/P_{cr} - 1} + \frac{z_{min} \lambda_0}{\pi w_0^2}}$$

**Comments**

1. If $P < P_{cr}$, $z_{SF}$ is imaginary $\Rightarrow$ No self-focusing!

   Competing effect: diffraction

2. Detrimental effect of self-focusing:



Focus at a point with extremly high intensity

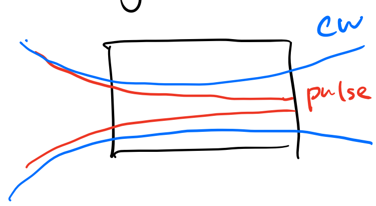
$\Rightarrow$ breakdown, optical damage.

3. Critical threshold depends on power, not intensity.

4. Only occurs when $n_2 > 0$, negative $n_2$ causes beam defocusing.

5. How to prevent? Tune the $z_{min}$! Can have negative $z_{min}$, $z_{SF}$

6. Applications: Kerr lens mode-locking.




## 3. Self-trapping (spatial soliton)

$$\text{Beam diffraction} \xleftrightarrow{\text{balance}} \text{Self-focusing.}$$


$$\theta_{diff} = \theta_{SF}.$$

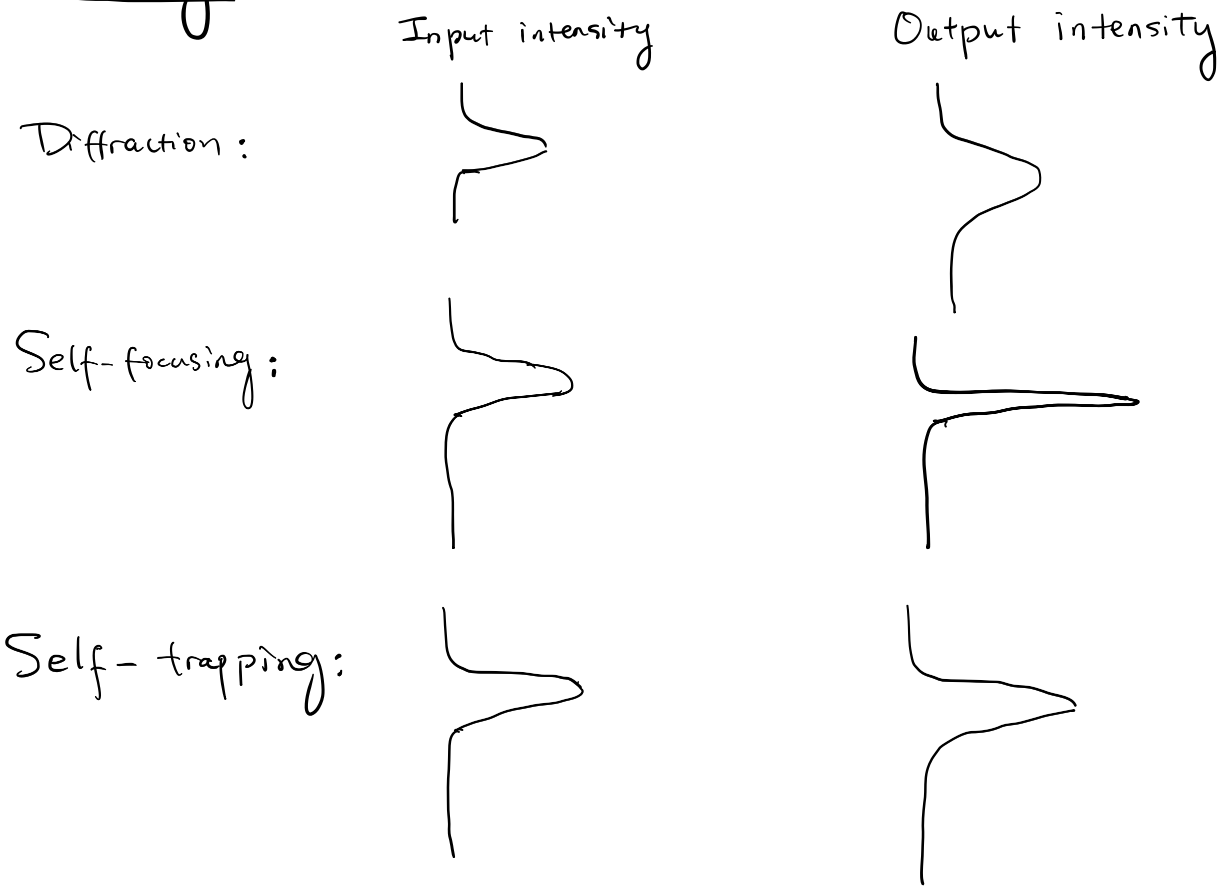
$$\Rightarrow \frac{0.61\lambda_0}{n\,d} = \sqrt{\frac{2n_2 I}{n_0}}$$

$d$: beam diameter

$$\Rightarrow I_{cr} = \frac{(0.61)^2 \lambda_0^2}{2 n_2 n_0 d^2} \quad \text{(critical intensity for self-trapping)}$$

$$P_{cr} = \left(\frac{\pi}{4}\right) d^2 \cdot I_{cr} = \frac{\pi (0.61)^2 \lambda_0^2}{8 n_0 n_2} \simeq \frac{\lambda_0^2}{8 n_0 n_2}$$

## Summary:


Input intensity
Output intensity
Diffraction:
Self-focusing:
Self-trapping: