Lecture 7. Optical waveguide II

Last lecture: ① TE mode in slab waveguide

② Waveguide modes and modes cut-off.

Today: ① TM modes in symmetric slab waveguide.

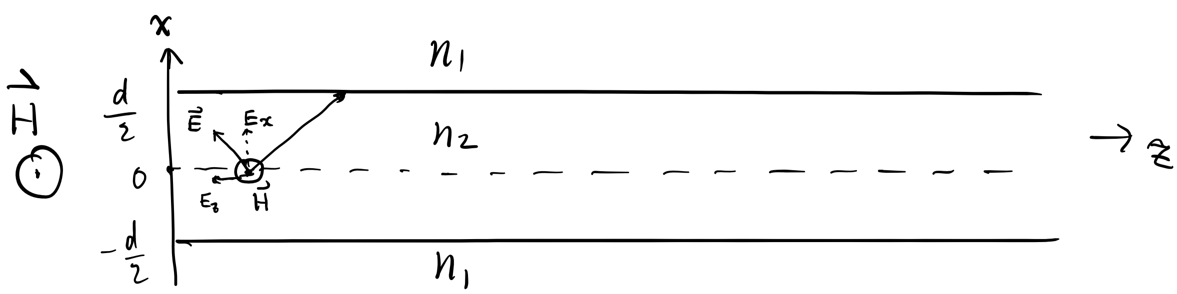
② Effective index

③ Effective index theory.

④ Orthogonality of waveguide modes

⑤ Waveguide dispersion

1 Guide TM modes in symmetric slab waveguide



TM mode:

$$\begin{cases} H_y \neq 0, \ H_x = 0, \ H_z = 0 \\ E_y = 0, \ E_x \neq 0, \ E_z \neq 0 \end{cases}$$

Field amplitutes:

$$\begin{cases} H_y(x, z, t) = H_m(x) e^{i(\omega t - \beta z)} \\ E_x(x, z, t) = \frac{i}{\omega\mu} \frac{\partial}{\partial z} H_y \\ E_z(x, z, t) = -\frac{i}{\omega\mu} \frac{\partial}{\partial x} H_y \end{cases}$$

To have confined TM wave in the slab waveguide:

$$H_m(x) = \begin{cases} A \sinh hx + B \cosh hx & |x| < \frac{d}{2} \\ C \exp(-qx) & x > \frac{d}{2} \\ D \exp(qx) & x < -\frac{d}{2} \end{cases}$$

B.C. H_y and E_z must be continous at $x = \pm \frac{d}{2}$,

$$\Rightarrow \quad h \tan\left(\tfrac{1}{2} h d\right) = \frac{n_2^2}{n_1^2}\, q \qquad \text{(even modes)}$$

$$h \cot\left(\tfrac{1}{2} h d\right) = -\frac{n_2^2}{n_1^2}\, q \qquad \text{(odd modes)}$$

$\Rightarrow$ combine :

$$\tan(hd) = \frac{2h\bar{q}}{h^2 - \bar{q}^2}, \quad \text{where} \quad \bar{q} = \frac{n_2^2}{n_1^2}\, q$$

very similar to TE, $\tan(hd) = \dfrac{2hq}{h^2 - q^2}$.

mode can also be solved by graphic solutions.

2- Effective index

Now, we have many guided mode (TE_0, TE_1, $TE_2 \cdots TM_0, TM_1 \cdots$). How to describe and compare each mode?

Recall: each mode has a unique set of u, v or h, q, and

$$\begin{cases} h = \sqrt{k_0^2 n_2^2 - \beta^2} = \sqrt{\left(\frac{n_2 \omega}{c}\right)^2 - \beta^2} \\ q = \sqrt{\beta^2 - k_0^2 n_1^2} = \sqrt{\beta^2 - \left(\frac{n_1 \omega}{c}\right)^2} \end{cases}$$

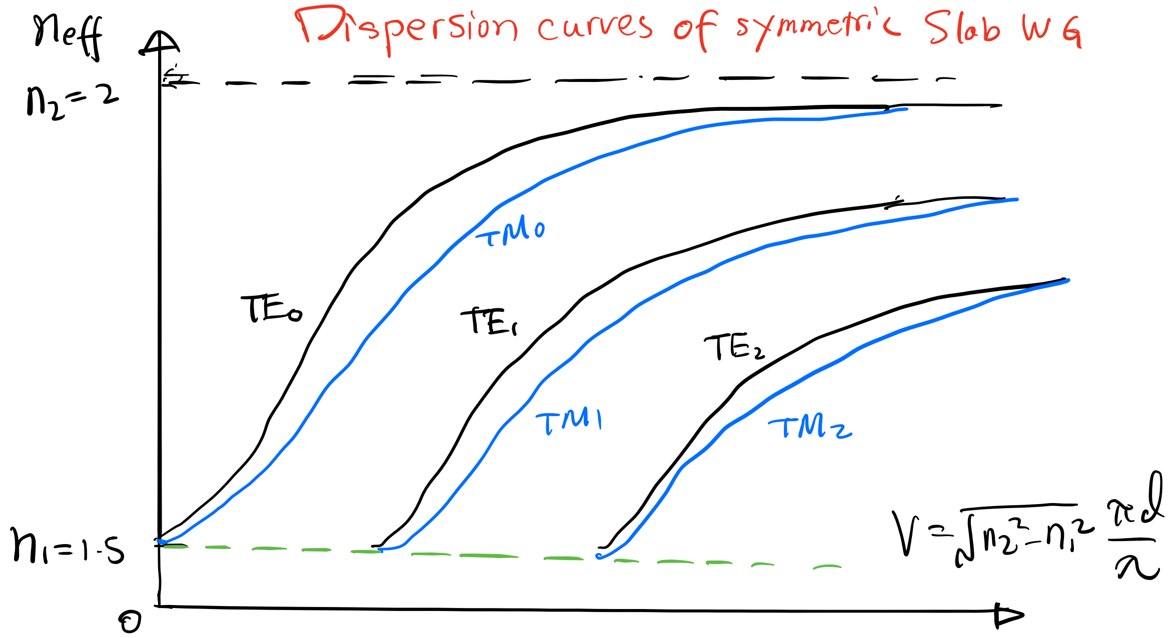
Therefore, each mode has a unique propagation constant β.

And we know that for guide wave, we must satisfy

$$k_0^2 n_1^2 < \beta^2 < k_0^2 n_2^2$$

Let's define $\beta = n_{eff} \cdot k_0$, so $n_1 < n_{eff} < n_2$ (effective index)

Dispersion curves of symmetric Slab WG



Comments:

① n_{eff} can be regarded as an "averged index" of waveguide (n_2) and cladding (n_1), it contains the information of

(1) mode confinement (larger n_{eff}, better confinement)

(2) dispersive preperty of material

$n_1(\lambda)$, $n_2(\lambda)$ $\rightarrow$ $n_{eff}(\lambda)$

(3) phase velocity of light in WG

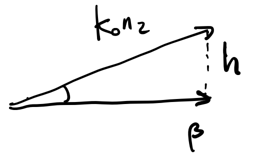
$$n_{eff} = \frac{c}{v_p}$$

② There's no mode cut-off for TE_0, TM_0 for symmetric slab waveguide. But for non-symmetric slab waveguide, there's cut-off! Please read Yariv P119 ~ 125.

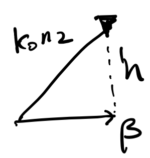
③ $r\uparrow$, $n_{eff}\uparrow$.

Physical meaning? $\lambda\downarrow$ or $d\uparrow$, better confinement.

④ higher order modes have less n_{eff} (β) or confinement



fundamental mode

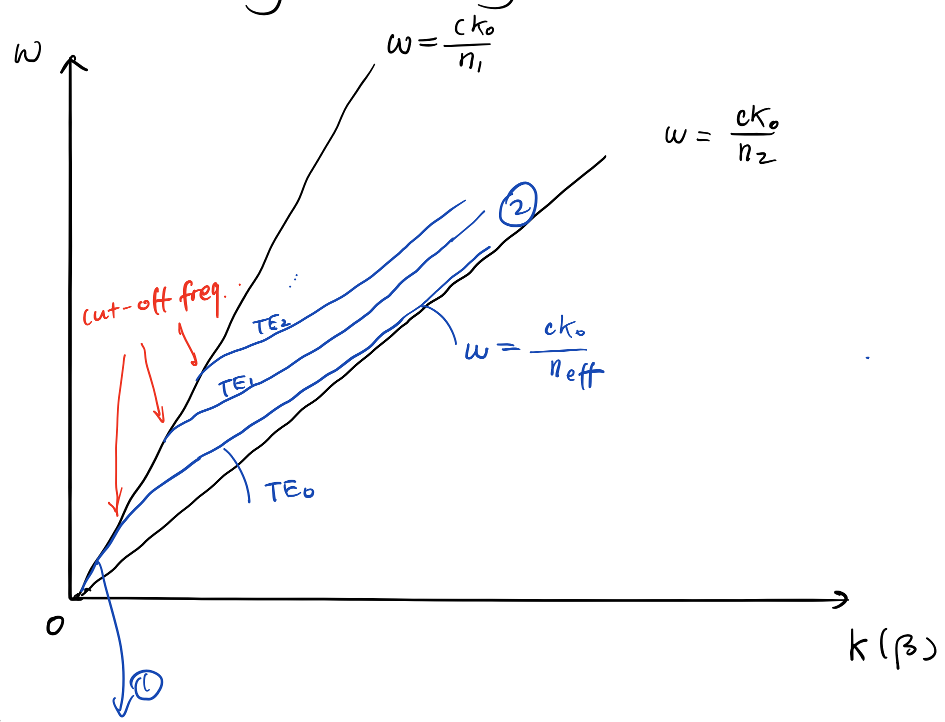


higher order mode

how to get rid of higher order modes ?

Bent waveguide, make total internal reflection harder !

A more common way of drawing waveguide modes.



Comments:

1 Region (1): When ω is very small (λ is very large), light can "see" more of the cl ding(n_1), so waveguide dispersion is very close to t dispersion in cladding (n_1)

2 Region (2): When ω is large (λ is small), the light is very confined in the waveguide. So the wavguide dispersion is very close to the dispersion in the "core" (n_2)

3 Even without material dispersion (n_1 n_2 are const.) waveguide mode has dispersion

$\Rightarrow$ Dispersion engineering

4 Each confined mode has its own n_{eff}, phase velocity, group velocity.

Fundamental mode has the highest n_{eff}.

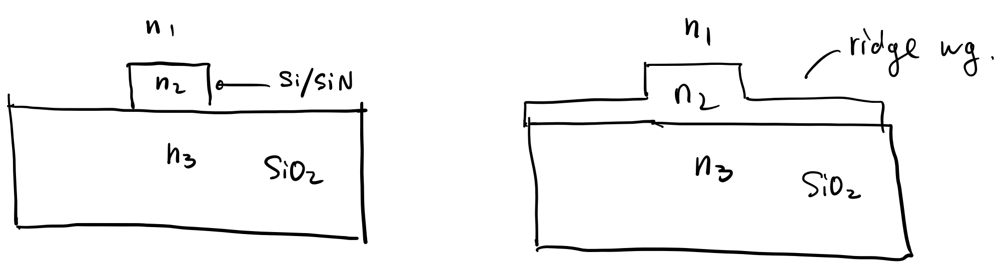
3. Effective index theory

Slab waveguide; wave (energy) is only confined in 1-D

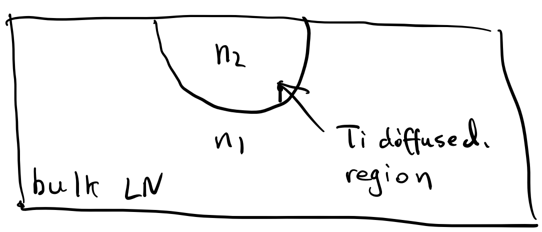
More commonly, we use 2-D waveguide with more complicated geometry (ridge, channel waveguide)

For example:

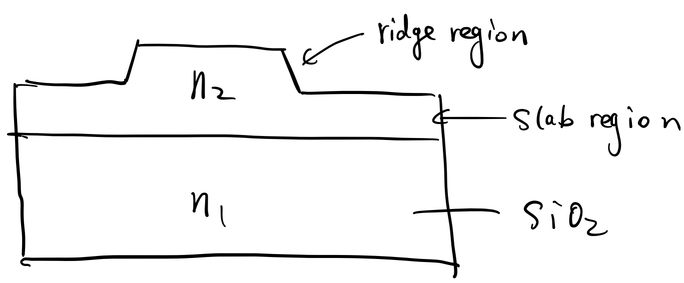
① Silicon/SiN photonics



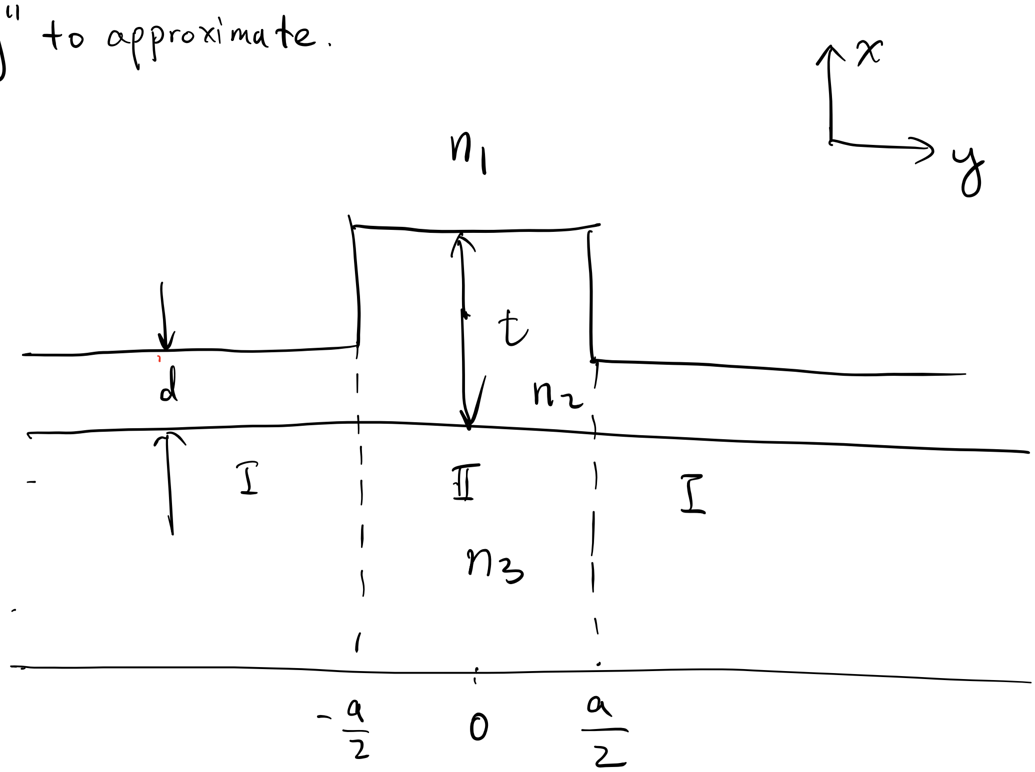
② Lithium niobate photonics (90s)



③ thin-film LN photonics



For those complicated waveguide geometries, no analytical Solutions are available. However, we can use "effective index theory" to approximate.



I: Asymmetric slab waveguide with thickness d.

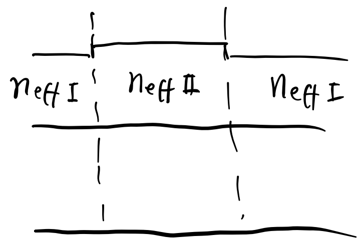
II ... . t.

$$n_{eff}(y) = \begin{cases} n_{eff\,I}, & y < -\frac{a}{2} \quad \text{(effective index of asymmetric slab I)} \\ n_{eff\,II} & -\frac{a}{2} < y < \frac{a}{2} \quad \text{(...... of .... II)} \\ n_{eff\,I} & \frac{1}{2}a < y, \quad \text{(... ... . I)} \end{cases}$$

Then, to calculate the neff of the 2-D confined guided mode, we just need to solve the Slab waveguide problem again along the y-direction, using $n_{eff\,I}$, $n_{eff\,II}$, $n_{eff\,I}$.

Case I ; Strong lateral waveguiding ($t \gg d$)

Case II : Weak lateral waveguiding ($t \sim d$)



4. Waveguide mode properties (Read P602. Yariv)

Orthogonality of eigenmodes :

For TE modes

$$\frac{\beta_m}{2\omega\mu} \iint (\vec{E}_m \cdot \vec{E}_n^*)\, da = \delta_{mn}$$

For TM mode

$$\frac{\beta_m}{2\omega} \iint \left(\vec{H}_m \cdot \frac{1}{\varepsilon} H_n^*\right) da = \delta_{mn}$$

normalization factor

Kronecker Delta

$$\begin{cases} \delta_{mn} = 1 & m = n \\ \delta_{mn} = 0 & m \neq o \end{cases}$$

5. Waveguide dispersion and dispersion engineering (P140. Yariv)

So far, we just discussed material dispersion (plane wave propagation in bulk)

wave eq. for confined wave in multi-mode waveguide.

$$\nabla_t^2 E_m(x,y) + \left(k_0^2 n^2(x,y)^2 - \beta^2\right) E_m(x,y) = 0 \qquad ①$$

transverse laplacian operator

Solution:

$$E(x,y,z,t) = E_m(x,y)\, e^{i(\omega t - \beta_m z)}, \qquad n_{eff} = \frac{\beta_m}{\omega/c}$$

Comments:

① For multi-mode WG, each mode has its distinct n_{eff}, v_g, v_p

$\Rightarrow$ modal dispersion.

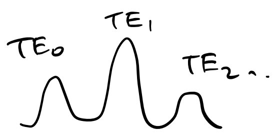
② β_m depends on waveguide geometry $n^2(x,y)$ and ω

$\Rightarrow$ waveguide dispersion

(reason: different $E_m(x,y)$ and ray paths at different freq.)

Consequence:

1) In multi-mode waveguide, if a sequence of pulses is represented by a superposition of modes,



the pulse will spread due to
1) modal dispersion
2) waveguide dispersion
3) material dispersion

2) In single-mode waveguide, the pulse spreading is due to
1) waveguide dispersion
2) material dispersion

Consider single-mode waveguide.

$$n_{eff} = n_{eff}\left[\, n_1(\omega),\ \ n_2(\omega),\ \omega \,\right]$$

Material dispersion

waveguide dispersion.

Using pertubation theory,

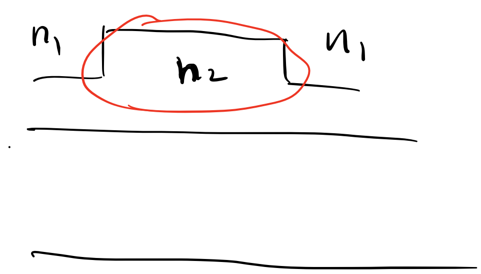
Apply a small $\delta\omega$ to wave-eq. ①, Let $\delta\beta_m^2$, δn_1^2, δn_2^2 be the corresponding variations, then

$$\delta\beta_m^2 = \Gamma_1 \frac{\omega^2}{c^2}\cdot\delta n_1^2 + \Gamma_2 \frac{\omega^2}{c^2}\delta n_2^2 + \Gamma_1 \frac{2\omega\delta\omega}{c^2} n_1^2 + \Gamma_2 \frac{2\omega\delta\omega}{c^2} n_2^2 \quad (2)$$

where Γ_1, Γ_2 are confinement factor given by

$$\Gamma_\alpha = \frac{\iint_{n_\alpha} E_m^* \cdot E_m \, dx.dy}{\iint_\infty E_m^* \cdot E_m \, dx.dy} \qquad (\alpha = 1,2)$$

eg: 2D- waveguide



Γ_2 measures how much energy is concentrated in n_2

Single-mode WG, drop m. eq. (2) can be written as

$$\frac{d\beta}{d\omega} = \Gamma_1 \frac{\omega}{c} \cdot \frac{n_1}{n_{eff}} \cdot \frac{\partial n_1}{\partial \omega} + \Gamma_2 \frac{\omega}{c} \cdot \frac{n_2}{n_{eff}} \cdot \frac{\partial n_2}{\partial \omega} + \frac{1}{c n_{eff}} \left(\Gamma_1 n_1^2 + \Gamma_2 n_2^2 \right)$$

plug in $\beta = n_{eff} \cdot \frac{\omega}{c}$.

$$\Rightarrow \frac{d n_{eff}}{d\omega} = \underbrace{\Gamma_1 \frac{n_1}{n_{eff}} \cdot \frac{\partial n_1}{\partial \omega} + \Gamma_2 \frac{n_2}{n_{eff}} \frac{\partial n_2}{\partial \omega}}_{\text{material dispersion}} + \underbrace{\frac{1}{\omega n_{eff}} \left(\Gamma_1 n_1^2 + \Gamma_2 n_2^2 - n_{eff}^2 \right)}_{\text{waveguide dispersion}}$$

$$\Rightarrow \frac{d n_{eff}}{d\omega} = \left(\frac{\partial n_{eff}}{\partial \omega} \right)_{material} + \left(\frac{\partial n_{eff}}{\partial \omega} \right)_{waveguide},$$

where $\left(\frac{\partial n_{eff}}{\partial \omega} \right)_{material} = \Gamma_1 \frac{n_1}{n_{eff}} \cdot \frac{\partial n_1}{\partial \omega} + \Gamma_2 \cdot \frac{n_2}{n_{eff}} \frac{\partial n_2}{\partial \omega},$

$$\left(\frac{\partial n_{eff}}{\partial \omega} \right)_{waveguide} = \frac{1}{\omega n_{eff}} \cdot \left(\Gamma_1 n_1^2 + \Gamma_2 n_2^2 - n_{eff}^2 \right)$$

If $n_1 \simeq n_2 = n$,

$$\left(\frac{\partial n_{eff}}{\partial \omega} \right)_{material} \simeq \frac{\partial n}{\partial \omega}$$

We can also write

$$\frac{dn_{eff}}{d\lambda} = \left(\frac{\partial n}{\partial \lambda}\right)_{mat.} + \left(\frac{\partial n_{eff}}{\partial \lambda}\right)_{wg.}$$

$$\frac{d^2 n_{eff}}{d\lambda^2} = \left(\frac{\partial^2 n}{\partial \lambda^2}\right)_{mat.} + \left(\frac{\partial^2 n_{eff}}{\partial \lambda^2}\right)_{wg.}$$

Consider the propagation of pulse in a single-mode waveguide of length L,

total phase shift. $\phi = n_{eff} \cdot \frac{\omega}{c} \cdot L$.

group delay: $\tau = \frac{d\phi}{d\omega} = L\left(\frac{n_{eff}}{c} + \frac{\omega}{c} \cdot \frac{dn_{eff}}{d\omega}\right)$

$$= L \cdot \left(\frac{n_{eff}}{c} - \frac{\lambda}{c} \cdot \frac{dn_{eff}}{d\lambda}\right)$$

The dispersion parameter D (ps/km·nm) in waveguide

$$D = \frac{d}{d\lambda}\left(\frac{\tau}{L}\right) = -\frac{1}{c\lambda}\left(\lambda^2 \frac{d^2 n_{eff}}{d\lambda^2}\right)$$

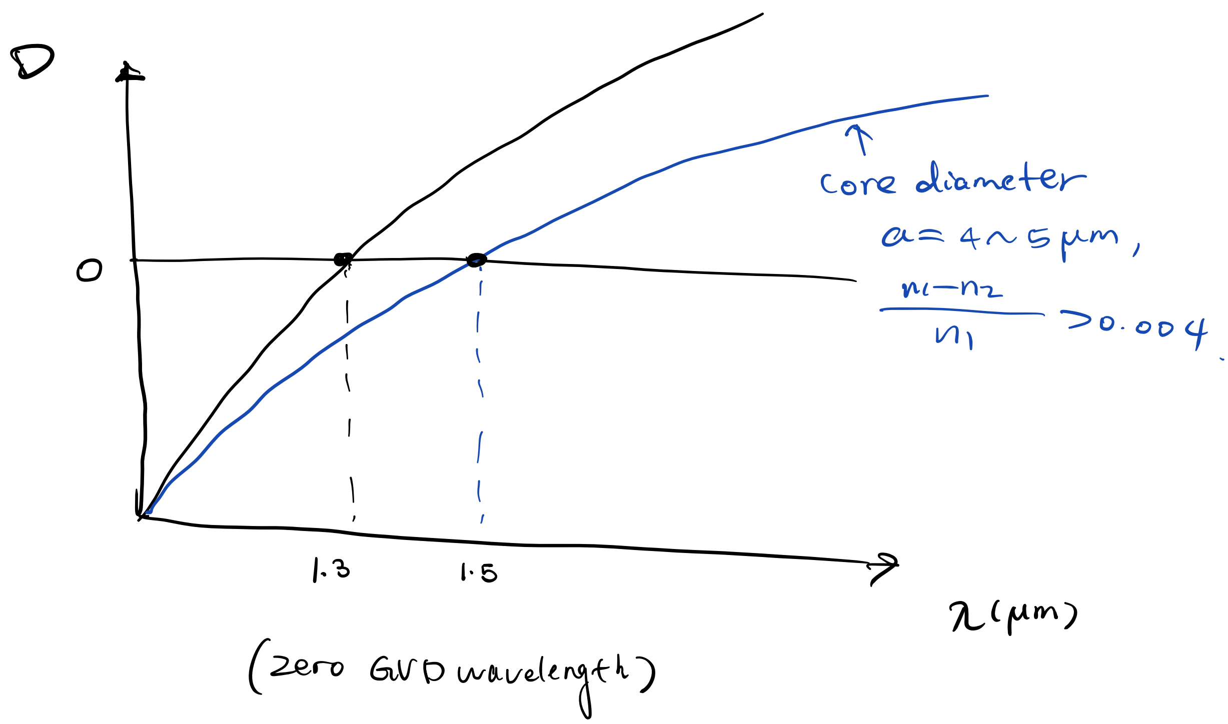
$$= -\frac{1}{c\lambda}\underline{\left(\lambda^2 \frac{\partial^2 n}{\partial \lambda^2}\right)_{mat}} - \frac{1}{c\lambda}\underline{\left(\lambda^2 \cdot \frac{\partial^2 n_{eff}}{\partial \lambda^2}\right)_{wg}}.$$

Comments:

① We can use waveguide dispersion to compensate material dispersion. (Dispersion engineering)

② Waveguide dispersion depends on n_1 n_2 and the core dimension.

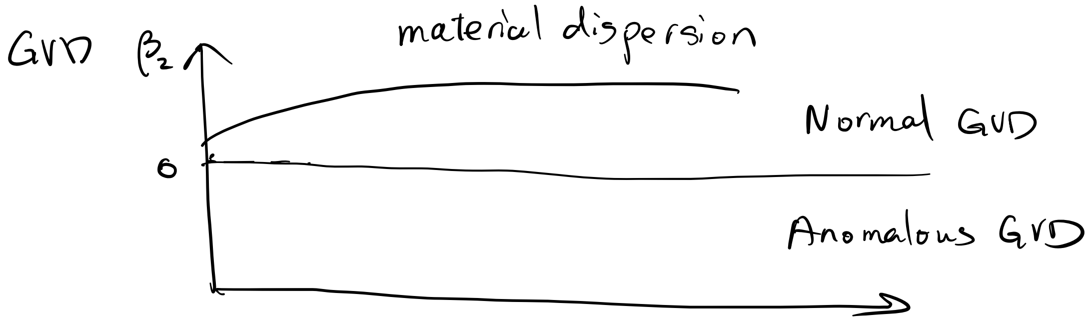
Eg. 1 Silica fiber



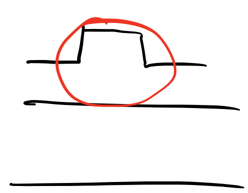
We can tailor the zero GVD wavelength to $1.5 \sim 1.6 \mu m$, where silica fiber has the lowest loss.

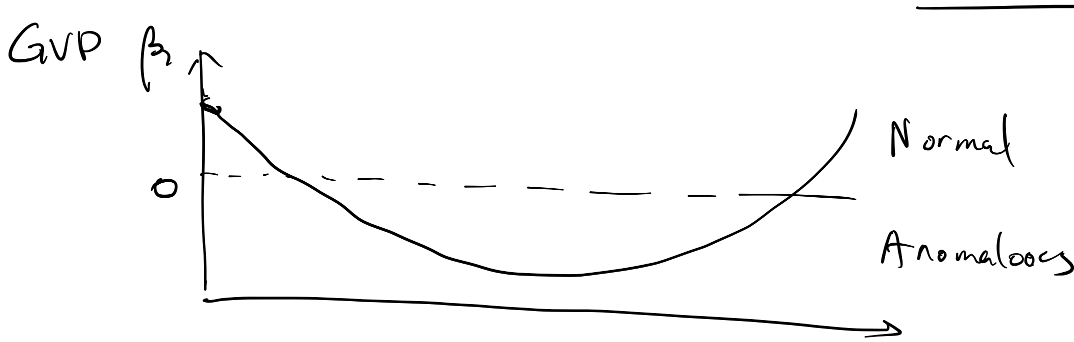
Eg 2: Broaden band zero GVD.

eg 3. Dispersion engineering in integrated photonics



$\Downarrow$ nanophotonic wg.





Applications : ① Soliton formation (Anomalous GVD with $\chi^{(3)}$)

② Supercontinuum generation (... ....)

③ Modulation instability . . . .