Lecture 6. Optical Waveguide I

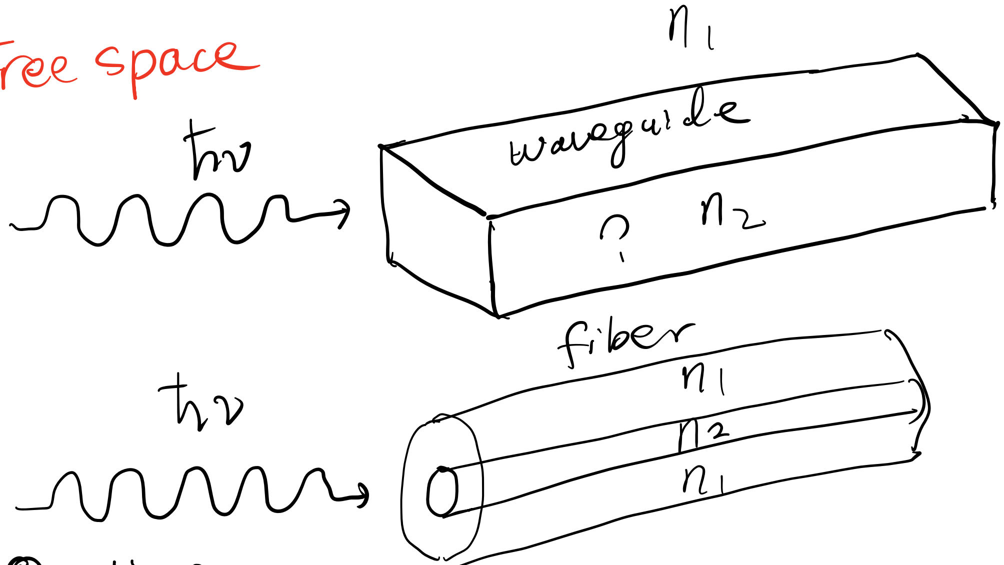
Learning objectives:

1. Symmetric slab waveguide $\begin{cases} \text{TE} \\ \text{TM} \end{cases}$
2. Waveguide modes and mode cut-off.
3. Effective index

So far, we have discussed wave propagation in free space, and at the interfaces of materials.

But in integrated photonics, fiber optics, we need to guide the light wave to manipulate the flow of light, and confine light tightly.

Free space

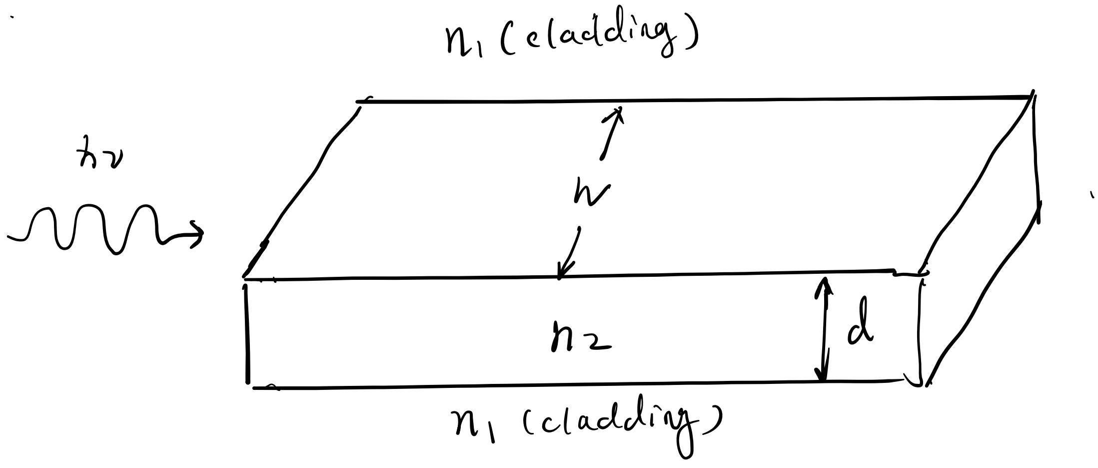


Questions:

1. What is the $\vec{E}$ distribution (Mode profile) in the WG?
2. what are the conditions to guide light waves?
3. How does the Waveguide tailor the dispersion, etc.

Answer: solve Maxwell equations in WGs, and impose proper boundary conditions.

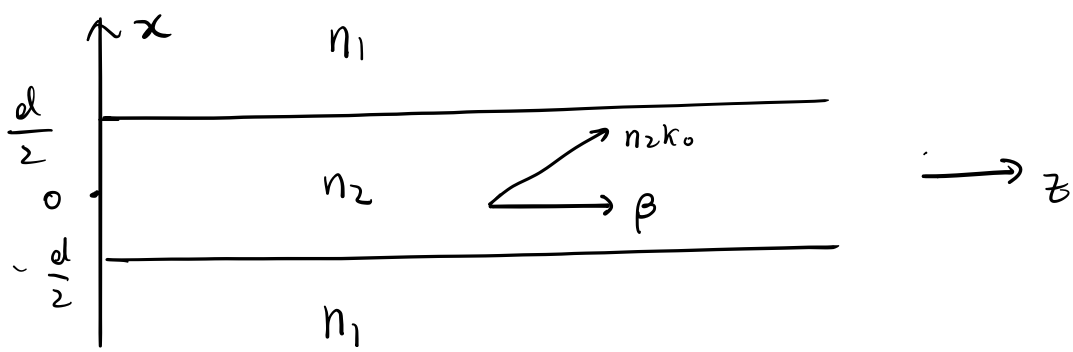
1. Symmetric slab waveguide



① Width (W) is infinite

② thickness (d) is finite.

2-D View:



① homogeneous along y, z

② $n(x) = \begin{cases} n_2, & |x| < \frac{d}{2} \\ n_1; & |x| > \frac{d}{2} \end{cases}$

Time-harmonic Maxwell equations:

$$\begin{cases} \nabla\times\vec{H} = i\omega\epsilon_0 n^2\vec{E} \\ \nabla\times\vec{E} = -i\omega\mu\vec{H} \end{cases}, \quad \text{where } n = \begin{cases} n_2 & |x|<\frac{d}{2} \\ n_1, & |x|>\frac{d}{2} \end{cases}$$

$$\Rightarrow \boxed{\nabla^2\vec{E} + n^2k_0^2\vec{E} = 0}$$

Assuming a wave (not plane wave) propagating along $\hat{z}$

$$\vec{E} = \underbrace{\vec{E}(x,y)}_{\text{not plane wave}} e^{i(\omega t-\beta z)}$$

$$\frac{\partial^2}{\partial x^2}E(x,y)\cancel{e^{i(\omega t-\beta z)}} + \frac{\partial^2}{\partial y^2}E(x,y)\cancel{e^{i(\omega t-\beta z)}} - \beta^2 E(x,y)\cancel{e^{i(\omega t-\beta z)}} + n^2k_0^2E(x,y)\cancel{e^{i(\omega t-\beta z)}} = 0$$

$$\Rightarrow \boxed{\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\vec{E}(x,y) + \left[k_0^2n^2(r)-\beta^2\right]\vec{E}(x,y) = 0} \quad (1)$$

Comment:

(1) this eq. applies to all layers.

(2) when $n^2k_0^2-\beta^2>0$, $\Rightarrow$ guided (confined) wave

(3) when $n^2k_0^2-\beta^2<0$, $\Rightarrow$ evanescent wave

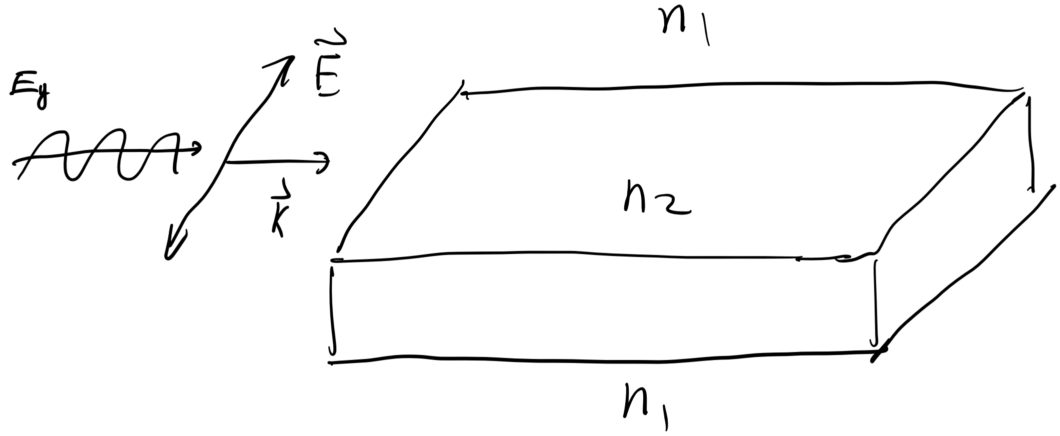
(E-field is decaying)

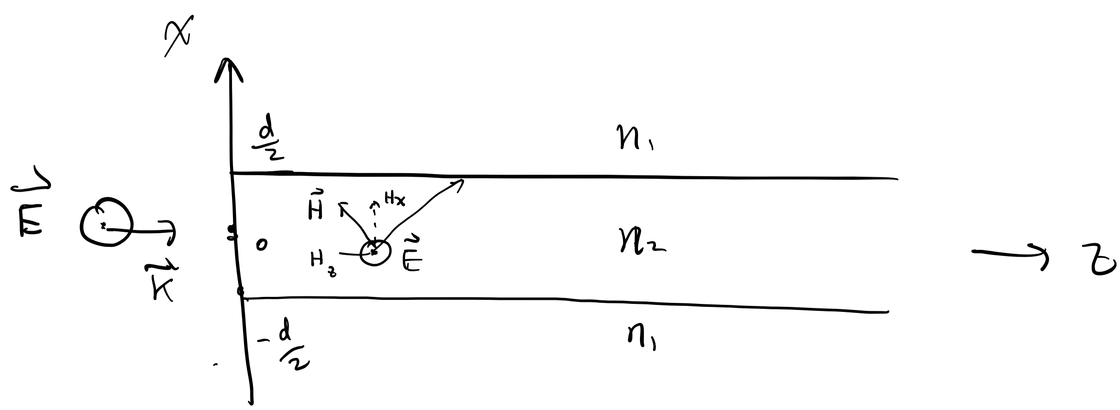
④ β is called propagation constant.

very important parameter, it determines whether the field is sinusoidally varying or exponentially decaying.

⑤ To solve $E(x,y)$, we need boundary conditions.

Guide TE Modes





TE Mode:

$$\begin{cases} E_y \neq 0, & E_x = 0, & E_z = 0 \\ H_y = 0, & H_x \neq 0, & H_z \neq 0 \end{cases}$$

Electric field amplitude of guide TE mode is

$$E_y(x, z, t) = E_m(x)\, e^{i(\omega t - \beta z)}$$

To satisfy eq. ①,

$$E_m(x) = \begin{cases} A \sin hx + B \cos hx, & |x| < \frac{d}{2} \quad \text{(Standing wave along } x\text{)} \\ C \exp(-qx), & x > \frac{d}{2} \quad \text{(Evanescent wave along } x\text{)} \\ D \exp(qx), & x < -\frac{d}{2} \quad (\text{.. --- .. ..}) \end{cases}$$

(Very similar to square-well potential in QM!)

Also, to satisfy eq ①, we must have

$$\begin{cases} h = \sqrt{k_0^2 n_2^2 - \beta^2} = \sqrt{\left(\frac{n_2 \omega}{c}\right)^2 - \beta^2} \\ q = \sqrt{\beta^2 - k_0^2 n_1^2} = \sqrt{\beta^2 - \left(\frac{n_1 \omega}{c}\right)^2} \end{cases}$$

So, to have propagating wave in n_2,

h, q have to be real.

$$\Rightarrow \boxed{n_1 k_0 < \beta < n_2 k_0}$$

Now, we need to determine A.B.C.d by B.C:

$$\begin{cases} \text{① } E_y \text{ must be continous at interfaces} \\ \text{② } H_z = \frac{i}{\omega\mu}\left(\frac{\partial E_y}{\partial x}\right) \text{ must be continous,} \end{cases}$$

$\Rightarrow$ 2 interfaces, 4 equations, 4 variables.

$$\begin{cases} A\sin(\frac{1}{2}hd) + B\cos(\frac{1}{2}hd) = C\exp(-\frac{1}{2}qd) & (\text{top } \vec{E_y}) \\ hA\cos(\frac{1}{2}hd) + hB\sin(\frac{1}{2}hd) = -qC\exp(-\frac{1}{2}qd) & (\text{top } \vec{H_z}) \\ -A\sin(\frac{1}{2}hd) + B\cos(\frac{1}{2}hd) = D\exp(-\frac{1}{2}qd) & (\text{bottom } \vec{E_y}) \\ hA\cos(\frac{1}{2}hd) + hB\sin(\frac{1}{2}hd) = qD\exp(-\frac{1}{2}qd) & (\text{bottom } \vec{H_z}) \end{cases}$$

$$\Rightarrow \begin{cases} 2A\sin(\frac{1}{2}hd) = (C-D)\exp(-\frac{1}{2}qd) & ① \\ 2hA\cos(\frac{1}{2}hd) = -q(C-D)\exp(-\frac{1}{2}qd) & ② \\ 2B\cos(\frac{1}{2}hd) = (C+D)\exp(-\frac{1}{2}qd) & ③ \\ 2hB\sin(\frac{1}{2}hd) = q(C+D)\exp(-\frac{1}{2}qd) & ④ \end{cases}$$

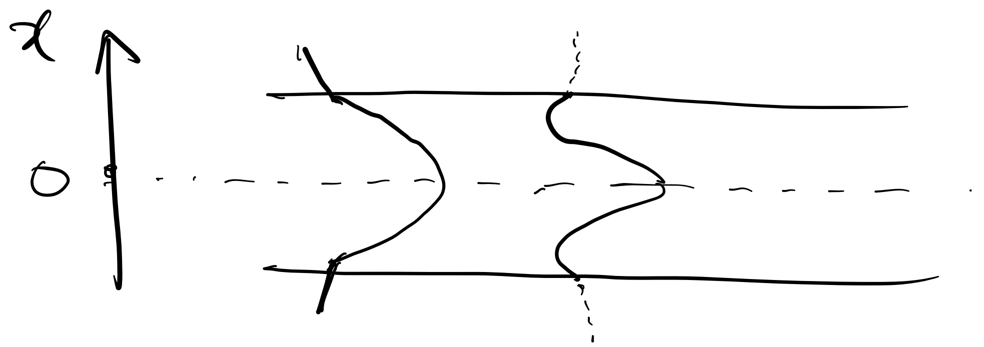
By examining the above eq, there're two sets of solutions:

① $A=0$, $C=D$, ①, ② are gone,

$$h\tan(\tfrac{1}{2}hd) = q \quad ①$$

$$E_m(x) = B\cos hx \rightarrow \text{symmetric TE}$$

Cos hx is an even function



2 $B=0$, $C=-D$, 3, 4 are gone

$$h\cot\left(\tfrac{1}{2}hq\right) = -q \quad 2$$

$E_m(x) = A\sin hx \rightarrow$ Asymmetric TE

What's the condition for having guided TE mode?

We can combine the two scenarios by

$$\left[h\tan\left(\tfrac{1}{2}hd\right) - q\right]\left[h\cot\left(\tfrac{1}{2}hd\right) + q\right] = 0$$

$$\Rightarrow \boxed{\tan(hd) = \frac{2hq}{h^2 - q^2}}$$

Let $u = \frac{1}{2}hd$, $v = \frac{1}{2}qd$, ① becomes

$$\boxed{u \tan u = v}$$

$\omega = 2\pi \frac{c}{\lambda}$

and

$$u^2 + v^2 = \left(\tfrac{1}{2}hd\right)^2 + \left(\tfrac{1}{2}qd\right)^2 = (n_2^2 - n_1^2)\left(\frac{\omega d}{2c}\right)^2$$

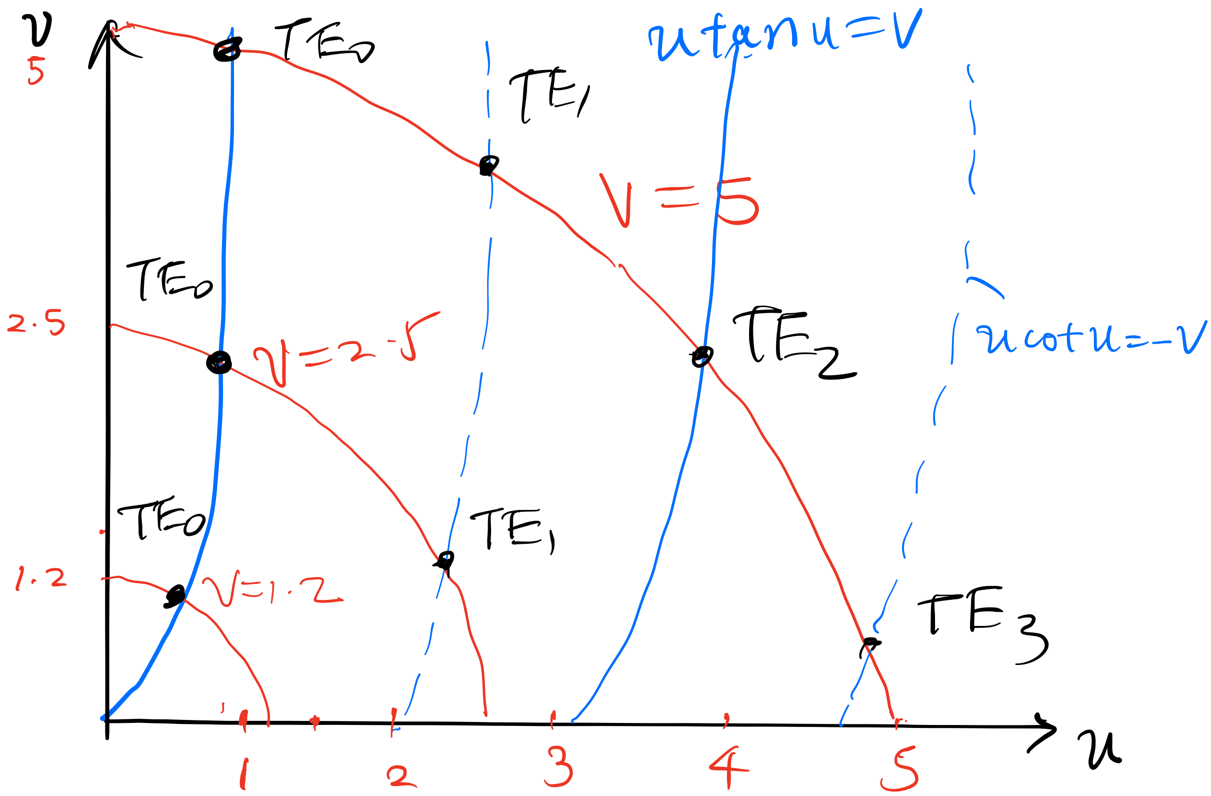
$$= (n_2^2 - n_1^2)\left(\frac{\pi d}{\lambda}\right)^2 \equiv V^2$$ (normalized frequency)

So, TE mode exist when:

$$\begin{cases} u \tan u = v \\ u^2 + v^2 = (n_2^2 - n_1^2)\left(\frac{\pi d}{\lambda}\right)^2 \equiv V^2 \end{cases}$$

circle.

How to solve? Graphic solution!!

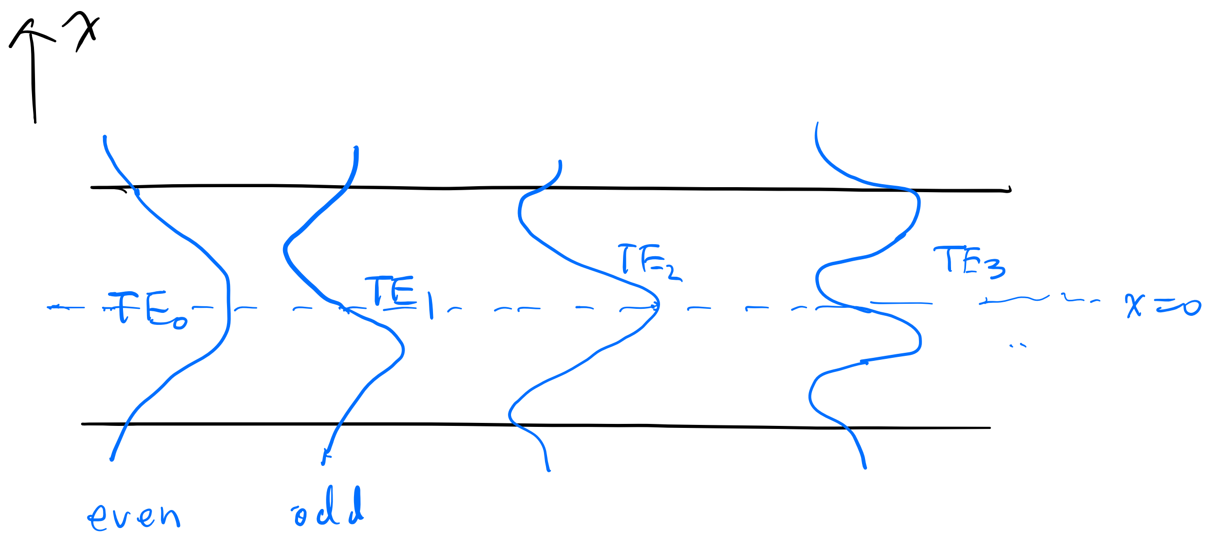


($n_1 = 1.5$, $n_2 = 1.6$, $\lambda = 1.55$, $d = 5\,\mu m$)

when $V = 1.2$, only TE_0 exists

when $V = 2.5$, TE_0 and TE_1 exist.

when $V = 5$, TE_0, TE_1, TE_2, TE_3 exist.



x
TE_0
TE_1
TE_2
TE_3
$x=0$
even
odd

Comments:

$$V^2 = (n_2^2 - n_1^2)\left(\frac{\pi d}{\lambda}\right)^2$$

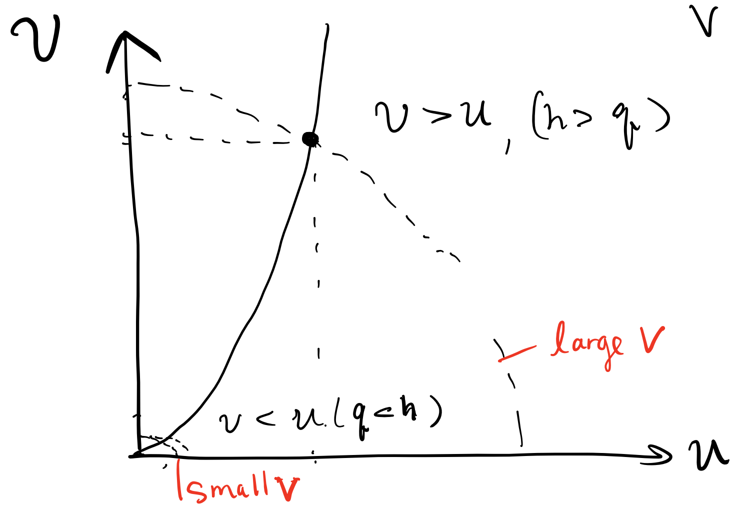
① increasing index contrast, thickness and decreasing λ will increase V, allowing more modes to propagate

② Mode cut-off? No cut-off for TE_0!

③ Single-mode?

Let's think about some extreme cases!

① Let's keep λ, n_1 unchanged, but make d very small



$$V^2 = (n_2^2 - n_1^2)\left(\frac{\pi d}{\lambda}\right)^2$$

very small.

$$u = \frac{1}{2} h d$$

$$v = \frac{1}{2} q d$$

As $d\downarrow$, $V\downarrow$, both h, q decreases

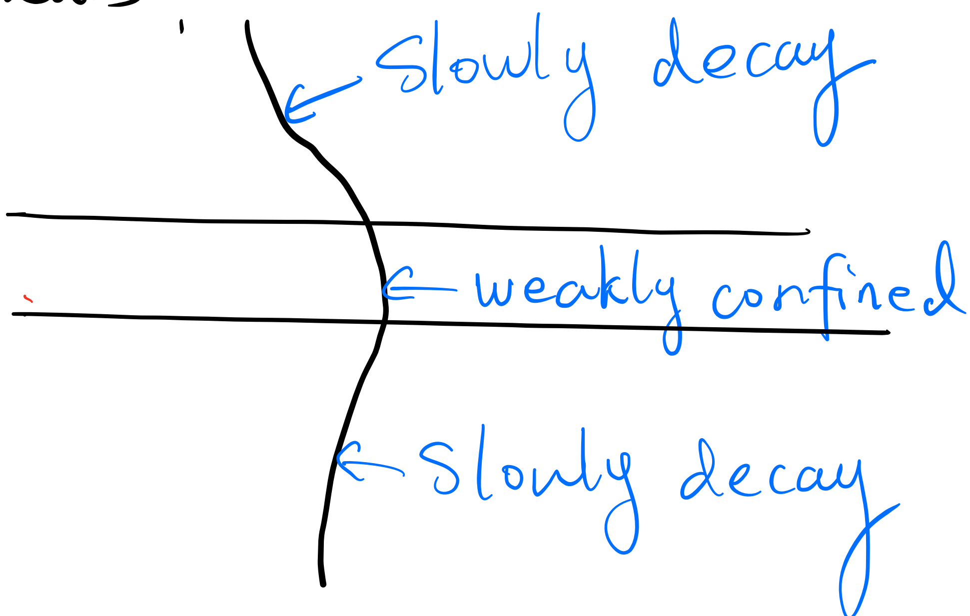
q decreases faster.

$\Rightarrow$ $\boxed{h, q \text{ are both very small}}$

note, inside the WG, $E_m \propto \cos(hx)$

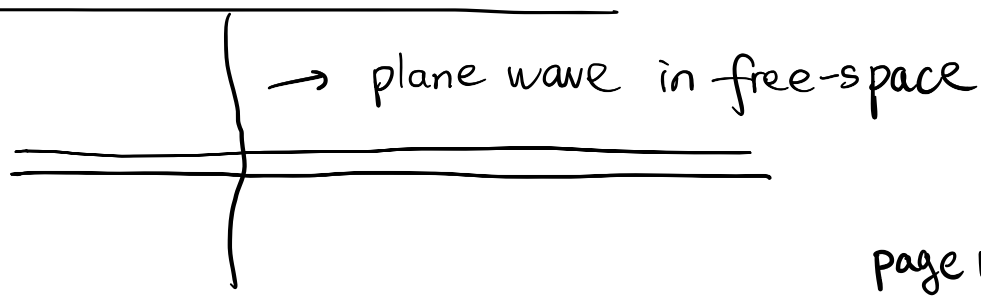
inside the cladding, $E_m \propto \exp(-qx)$.

This means :



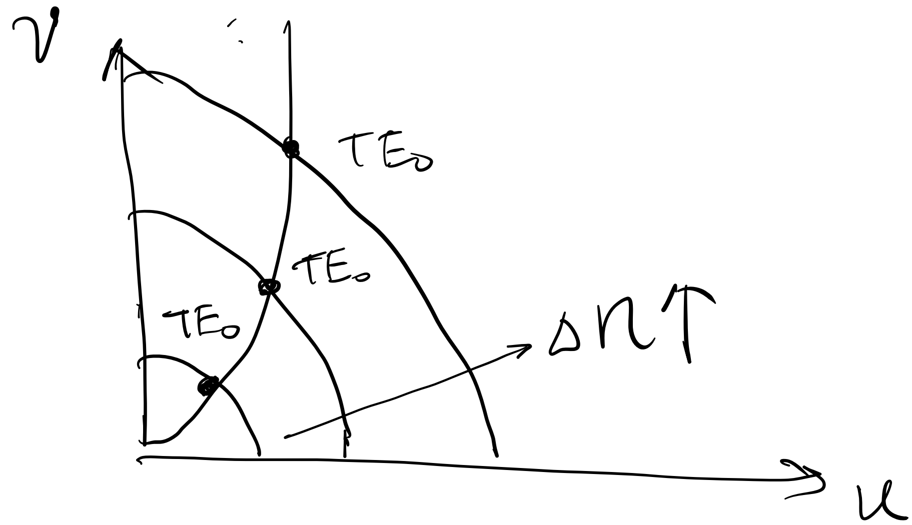
when d is small, wave is weakly guided in the waveguide.

The most extreme case :



② Let's make index contrast bigger!

$$\Delta n = n_2 - n_1 \uparrow$$



Any difference between these TE_0 modes?

Better confinement!