Lecture 6. $\chi^{(2)}$ nonlinear optical processes (DFG, OPA, DOPA)

Learning objectives:

① DFG

② Optical parametric amplification (OPA)

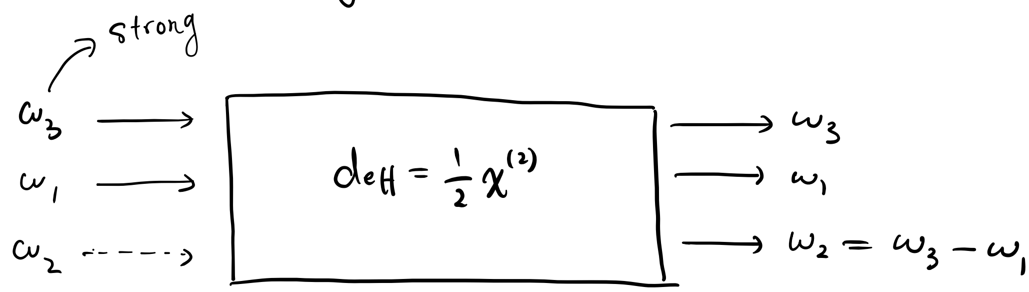
$\Big\}$ Same process two perspectives

③ Degenerate OPA (DOPA)

④ Squeezing

1. DFG and OPA

Problem to study:



Assumptions: ① ω_3 is a strong wave (undepleted), can be treated as a const.

② No input field at ω_2 (can be noise)

CAEs:

$$\begin{cases} \dfrac{dA_1}{dz} = \dfrac{2i\omega_1^2 d_{eff}}{k_1 c^2} \overset{\text{Const}}{A_3} \cdot A_2^* e^{i\overset{\Delta k = k_3 - k_1 - k_2}{\Delta k} z} & ① \\ \dfrac{dA_2}{dz} = \dfrac{2i\omega_2^2 d_{eff}}{k_2 c^2} A_3 \cdot A_1^* e^{i\Delta k z} & ② \end{cases}$$

When $\Delta k = 0$, $e^{i\Delta k z} = 1$

$$\frac{d}{dz} ② \Rightarrow \frac{d^2 A_2}{dz^2} = \frac{2i\omega_2^2 d_{eff}}{k_2 c^2} \cdot A_3 \cdot \frac{dA_1^*}{dz}$$ plug in ①

$$\Rightarrow \frac{d^2 A_2}{dz^2} = \frac{4\omega_1^2\omega_2^2 d_{eff}}{k_1 k_2 c^4} A_3 \cdot A_3^* \cdot A_2 \equiv \overset{\text{Coupling coeff.}}{k} \cdot A_2 \quad ③$$

where $$\kappa^2 = \frac{4 d_{eff}^2 \cdot \omega_1^2 \omega_2^2}{k_1 k_2 c^4} |A_3|^2 .$$

General solution of ③ is

$$A_2(z) = C \sinh \kappa z + D \cosh \kappa z .$$

Boundary condition: $A_2(0) = 0$, $A_1(0)$ is arbitray

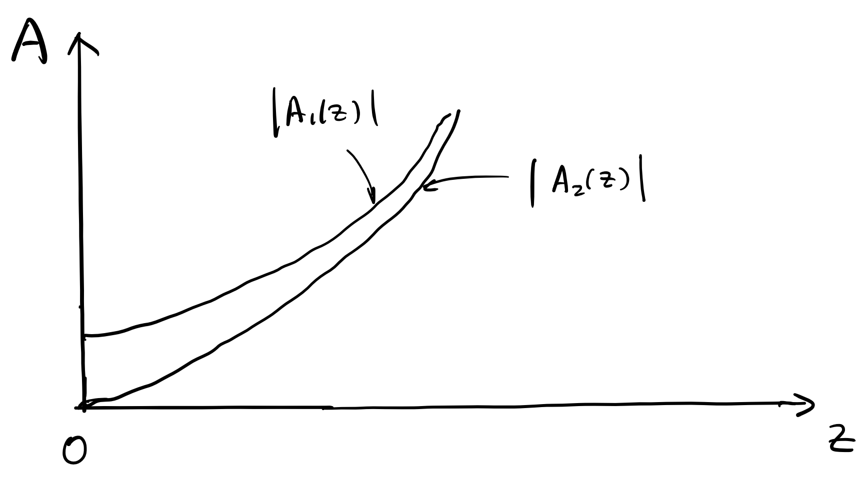
$$\Rightarrow A_2(0) = C \cdot \sinh \kappa \cdot 0 + D \cosh \kappa \cdot 0 = 0$$

$$\Rightarrow D = 0.$$

Solutions of ①. ② are:

$$\begin{cases} A_1(z) = A_1(0) \cosh \kappa z . \\ A_2(z) = i \left(\frac{n_1 \omega_2}{n_2 \omega_1} \right)^{1/2} \cdot \frac{A_3}{|A_3|} A_1^*(0) \sinh \kappa z \end{cases}$$ exponential growth!

ϕ_3



Comments:

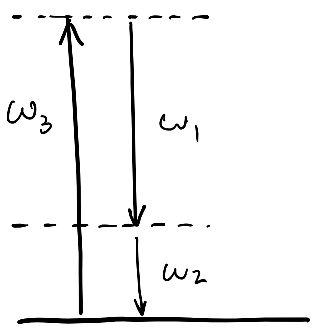
① For $kz \gg 1$, $|A_1(z)|$ and $|A_2(z)|$ both monotonically grow. (amplified). For SFG, oscillatory!

② ω_1 retains its initial phase.

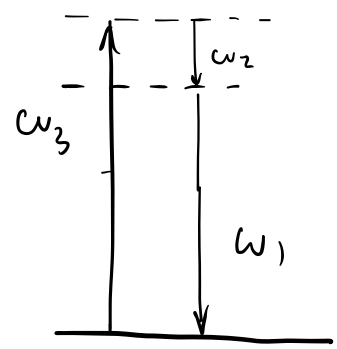
ω_2 has a phase that depends on ω_1 and ω_3

③ Power is conserved. pump (ω_3) will deplete as ω_1 and $\omega_2 \uparrow$

Physical Interpretation:



①



②

① the presence of ω_1 stimulates the downward transition, leading to the generation of ω_2 field

② ... ... ω_2 .. .. ω_1 ....

③ The generation of the ω_1 field reinforce the generation of ω_2, and vice versa, so each wave grows exponentially

Two perspectives on this process: ($\omega_3 - \omega_1 = \omega_2$)

① ω_1 — pump; ω_2 — output of interest; ω_3 — pump

$\boxed{\text{DFG}}$

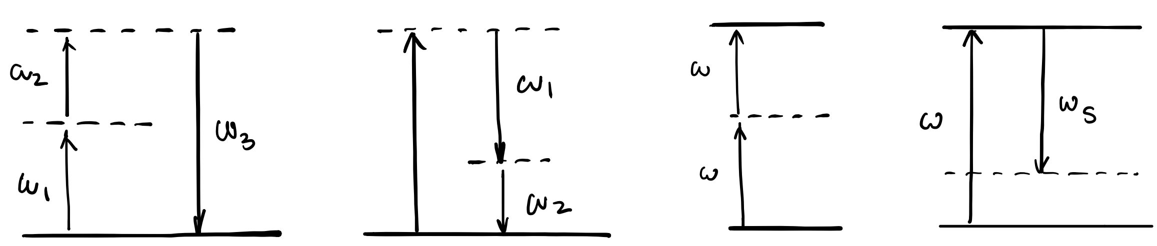
② ω_1 — signal amplified. ; ω_2 — idler; ω_3 — pump

$\boxed{\text{Parametric Amplification (OPA)}}$

Parametric process:

- Initial & Final quantum state of medium are identical.

e.g.



SFG or SHG | DFG / OPA | Two-photon absorption | Raman Amplification

$\underbrace{\text{SFG or SHG} \quad \text{DFG / OPA}}_{\text{Parametric process.}}$ $\quad$ $\underbrace{\text{Two-photon absorption} \quad \text{Raman Amplification}}_{\text{Non-parametric process.}}$

2. Optical parametric amplification (OPA)

What is the gain of the OPA over some distance (z)?

goal: calculate $\frac{|A_1(z)|^2}{|A_1(0)|^2}$

CAEs
$$\begin{cases} \frac{dA_1}{dz} = \frac{2i\omega_1^2 d_{eff}}{k_1 c^2} A_3 \cdot A_2^* e^{i\Delta k z} \\ \frac{dA_2}{dz} = \frac{2i\omega_2^2 d_{eff}}{k_2 c^2} A_3 \cdot A_1^* e^{i\Delta k z} \end{cases}$$

$\Delta k = k_3 - k_1 - k_2$

Generic solution:

$$\begin{cases} A_1(z) = \left[A_1(0)\left(\cosh gz - \frac{i\Delta k}{2g}\sinh gz\right) + \frac{\kappa_1}{g} A_2^*(0)\sinh gz\right] e^{i\frac{\Delta k z}{2}} \\ A_2(z) = \left[A_2(0)\left(\cosh gz - \frac{i\Delta k}{2g}\sinh gz\right) + \frac{\kappa_2}{g} A_1^*(0)\sinh gz\right] e^{i\frac{\Delta k}{2}} \end{cases}$$

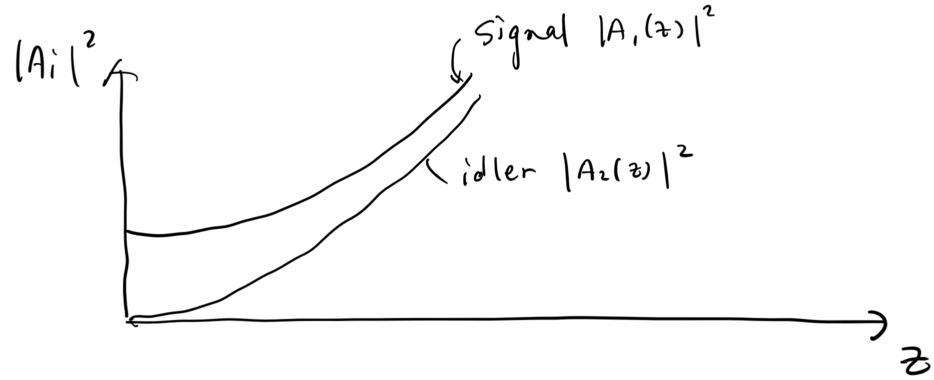
where $g = \sqrt{\kappa_1 \kappa_2^* - \left(\frac{\Delta k}{2}\right)^2}$. $\quad \kappa_i = \frac{2i\omega_i^2 d_{eff} \cdot A_3}{k_i c}$

① when $\Delta k = 0$, $A_2(0) = 0$ (no idler input)

$$A_1(z) = A_1(0)\cosh gz = A_1(0)\cdot\frac{e^{gz}+e^{-gz}}{2} = \frac{1}{2}A_1(0)\exp(gz)$$

$$A_2(z) = i\left(\frac{n_1\omega_2}{n_2\omega_1}\right)^{1/2}\cdot\frac{A_3}{|A_3|} A_1^*(0)\sin gz \sim A_1^*(0)\exp(gz)$$

$$\boxed{\text{Signal gain} = \frac{|A_1(z)|^2}{|A_1(0)|^2} \simeq \frac{1}{4}\exp(gz)^2} \quad g = \sqrt{\frac{4\omega_1^2\omega_2^2 d_{eff}^2 |A_3|^2}{k_1 k_2 \cdot c^2}}$$



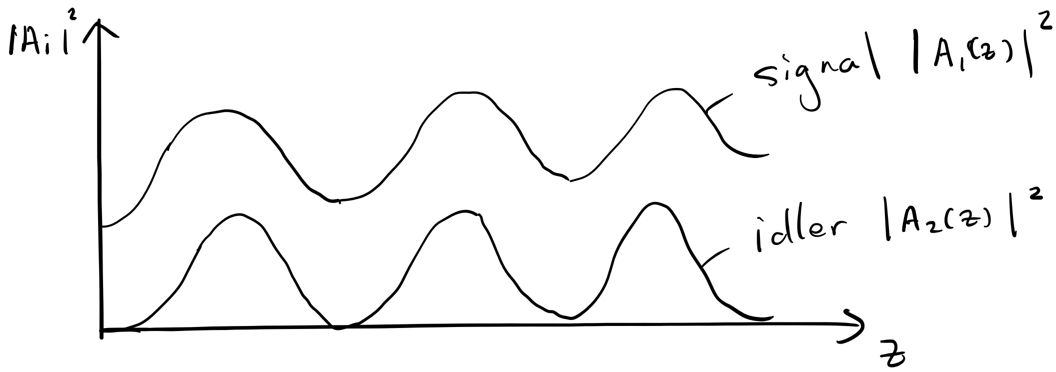
(2) When $\Delta k \neq 0$, $A_2(0) = 0$ (no idler input)

As $g = \sqrt{\kappa_1 \kappa_2^* - \left(\frac{\Delta k}{2}\right)^2}$,

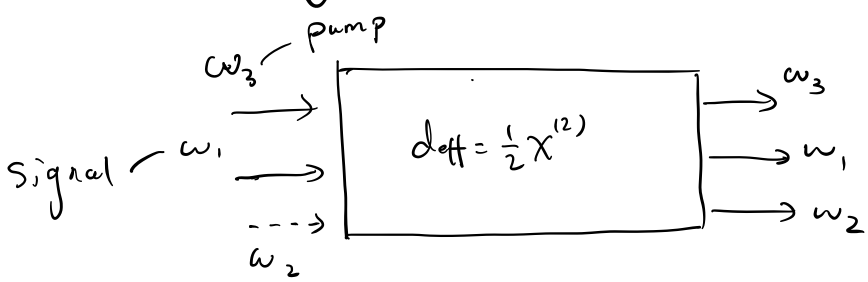
we need to compare $\kappa_1 \kappa_2^*$ with $\left(\frac{\Delta k}{2}\right)^2$

If $\kappa_1 \kappa_2^* > \left(\frac{\Delta k}{2}\right)^2$, amplification can still happen.

If $\kappa_1 \kappa_2^* < \left(\frac{\Delta k}{2}\right)^2$, oscillatory!



OPA tuning:

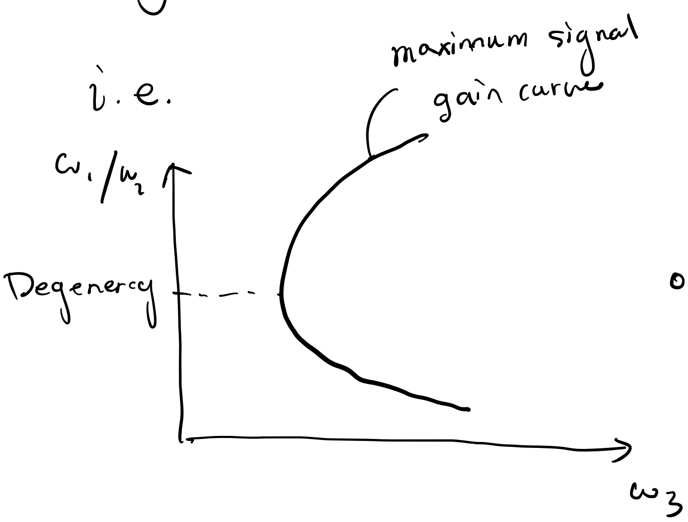


$$\begin{cases} \omega_1 + \omega_2 = \omega_3 \\ k_1 + k_2 = k_3 \\ n_1\frac{\omega_1}{c} + n_2\frac{\omega_2}{c} + n_3\frac{\omega_3}{c} \end{cases}$$

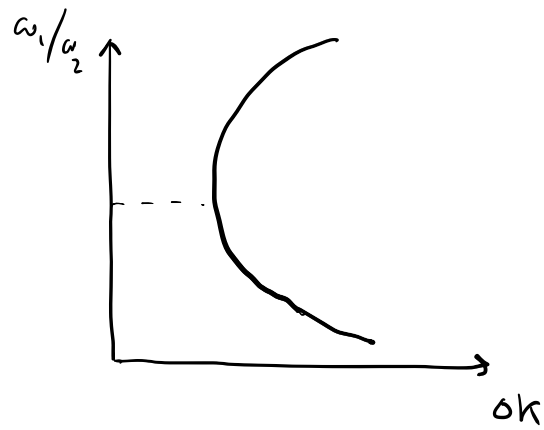
$g = \sqrt{k_1 k_2^* - \left(\frac{\Delta k}{2}\right)^2}$. when $\Delta k \uparrow$, $g \downarrow$.

However, since idler input is zero, or noise (phase is random) by adjusting ω_3, we can always satisfy $k_1 + k_2 = k_3$. ($\Delta k = 0$)

i.e.



or

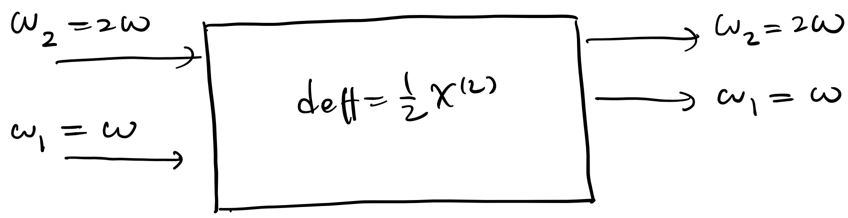


Comments:

For OPAs with zero idler input, by tuning the pump wavelength, or phase matching condition, we can tune the wavelength for maxium signal gain.

3. Degenerate OPA (DOPA)

A special operating point of OPA, where $\omega_3 = 2\omega_1 = 2\omega_2$



In this case, we can no longer assume zero idler input, since signal can be regarded as idler, and vice versa.

(Signal and Idler are indistinguishable!)

Assume $\Delta k = 0$

signal/idler $\quad \omega^2 \quad$ pump

CAE: $$\frac{dA_1}{dz} = \frac{2i\omega_1^2 d_{eff}}{k_1 c^2} A_2 \cdot A_1^*$$

$$\Rightarrow \frac{d^2 A_1}{dz^2} = kA_1$$

Solution: $A_1(z) = C\sinh(kz) + D\cos(kz)$

Boundary condition:

$$A_1(0) = D = A_{10} e^{\phi_{10}} \neq 0$$

$$\Rightarrow \quad A_1(z) = C \cdot \sinh(kz) + A_{10} e^{\phi_{10}} \cosh(kz)$$

$$\frac{dA_1(z)}{dz} = C\cdot K \cosh(Kz) + A_{10}e^{\phi_{10}} \sinh(\gamma z)$$

$$\left(\frac{dA_1(z)}{dz}\right)_{z=0} = C\cdot K = i\frac{\omega d_{eff}}{n_1 c} A_2(0)\cdot A_1^*(0)$$

$A_{20}e^{i\phi_{20}}$

$$\Rightarrow \quad C\cdot K = i\cdot \underbrace{\boxed{\frac{\omega d_{eff}}{n_1 c}}}_{g}\cdot A_{10}\cdot A_{20}{}^{i(\phi_{20}-\phi_{10})}$$

$$\Rightarrow A_1(z) = \left[i\frac{g}{\gamma}A_{10}A_{20}e^{i(\phi_{20}-\phi_{10})}\right]\sinh(\kappa z) + A_{10}e^{\phi_{10}}\cosh(\gamma z)$$

$i = e^{i\frac{\pi}{2}}$

$$= i A_{10}e^{i(\phi_{20}-\phi_{10})}\sinh(\kappa z) + A_{10}e^{i\phi_{10}}\cosh(\kappa z)$$

$$= A_{10}e^{i\phi_{10}}\Big[\cosh(\kappa z) + e^{i\overbrace{(\phi_{20}-2\phi_{10}+\frac{\pi}{2})}^{\delta}}\sinh(\gamma z)\Big]$$

Comments

① When $\delta = (\phi_{20} - 2\phi_{10} + \frac{\pi}{2}) = 0.$

$$A_1(z) = A_{10}e^{i\phi_{10}}e^{\kappa z} \quad \rightarrow \text{Amplification!}$$

② When $\delta = (\phi_{20} - 2\phi_{10} + \frac{\pi}{2}) = \pi$ (deamplification)

$$A_1(z) = A_{10}e^{i\phi_{10}}e^{-\kappa z} \quad \rightarrow \text{Attenuation}$$

③ Degenerate OPA is a phase sensitive amplifier!

4. Quadrature squeezing

Assume input field phase is unknown

$$A_1(z) = |A_1(0)| e^{i\phi_{10}} \left[\underbrace{\cosh(kz)}_{C} + e^{i(\phi_{20} - 2\phi_{10} + \frac{\pi}{2})} \underbrace{\sinh(kz)}_{S} \right]$$

$$\frac{A_1(z)}{|A_1(0)|} = Ce^{i\phi_{10}} + Se^{i(\phi_{20} - \phi_{10} + \frac{\pi}{2})}$$

① When $\phi_{20} = -\frac{\pi}{2}$:

$$\frac{A_1(z)}{|A_1(0)|} = Ce^{i\phi_{10}} + Se^{-i\phi_{10}} = \overbrace{(C+S)}^{e^{kz}}\cos\phi_{10} + i\overbrace{(C-S)}^{e^{-kz}}\sin\phi_{10}$$

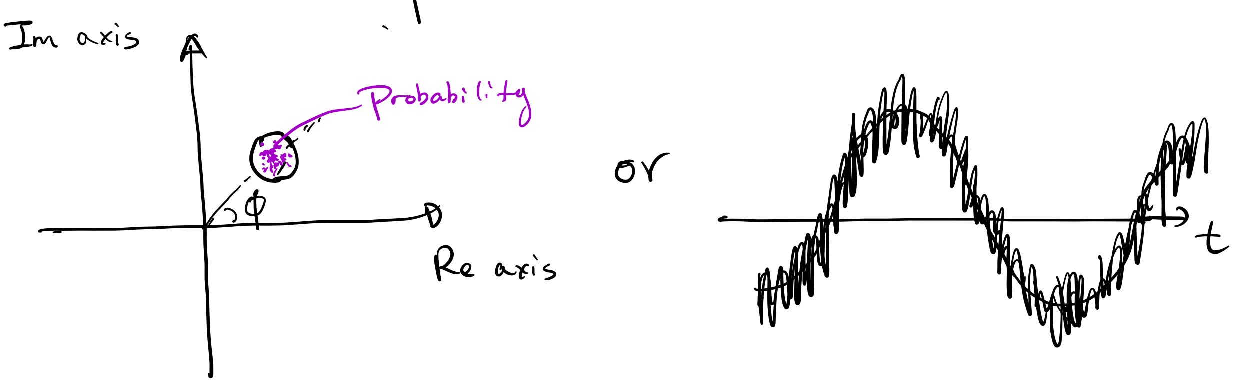
large enhancement of $\mathrm{Re}[A_1(z)]$.

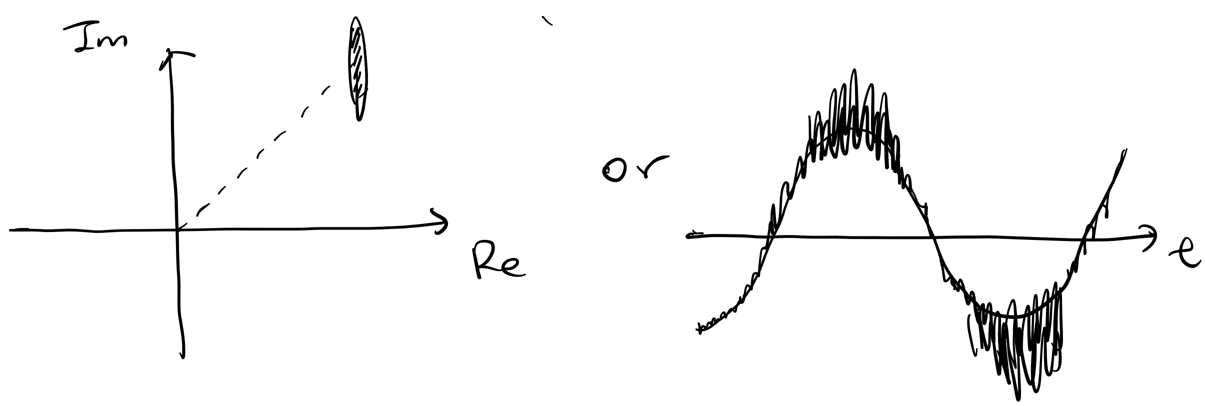
large reduction of $\mathrm{Im}[A_1(z)]$

② When $\phi_{20} = -\frac{\pi}{2}$, exactly opposite.

Suppose input field has fluctuations

Input field:





After the DOPA, noises are "pushed" to other quadrature