# Lecture 19: Laser mode-locking

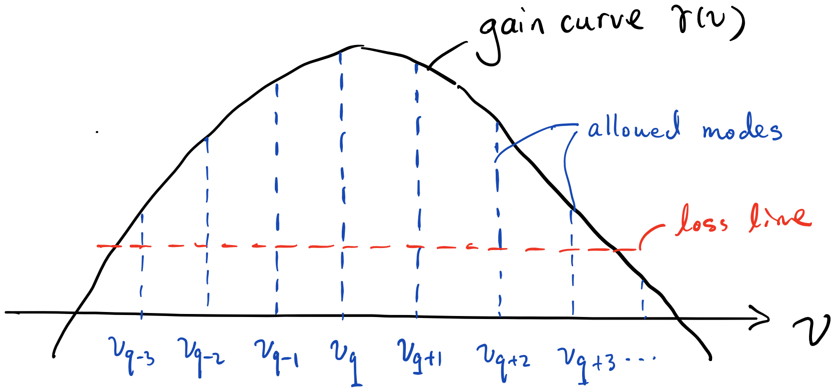
1. Mode-locking

2. Mode-locking techniques $\left\{ \begin{array}{l} \text{Active} \\ \text{Passive} \end{array} \right.$

3. Comparisoon between active and passive mode-locking

# 1. Mode-locking

Recall last lecture: Multi-mode lasing.




Mode spacing: $\nu_q - \nu_{q-1} = FSR = \dfrac{c}{2n\ell}$

$$\omega_q - \omega_{q-1} = 2\pi \cdot FSR = \frac{\pi c}{n\ell} \overset{n=1}{=} \frac{\pi c}{\ell} = \Omega$$

Total E-field:

$$E(t) = \sum_m C_m e^{i[(\omega_0 + m\Omega) + \phi_m]}$$

$C_m$: amplitude of $m_{th}$ mode

$\phi_m$: phase of $m_{th}$ mode.

If $\phi_m$ and $\Omega$ are fixed. (fixed phase and mode-spacing)

At $\tau = \frac{2\pi}{\Omega} = \frac{2l}{c}$

$$E(t+\tau) = \sum_m C_m \exp\left\{ i\left[(\omega_0 + m\Omega)\left(t + \frac{2\pi}{\Omega}\right) + \phi_m\right]\right\}$$

$$= \sum_m C_m \exp\left\{ i\left[(\omega_0 + m\Omega)t + \phi_m\right]\right\} \exp\left\{ i\left[2\pi\left(\frac{\omega_0}{\Omega} + m\right)\right]\right\}$$

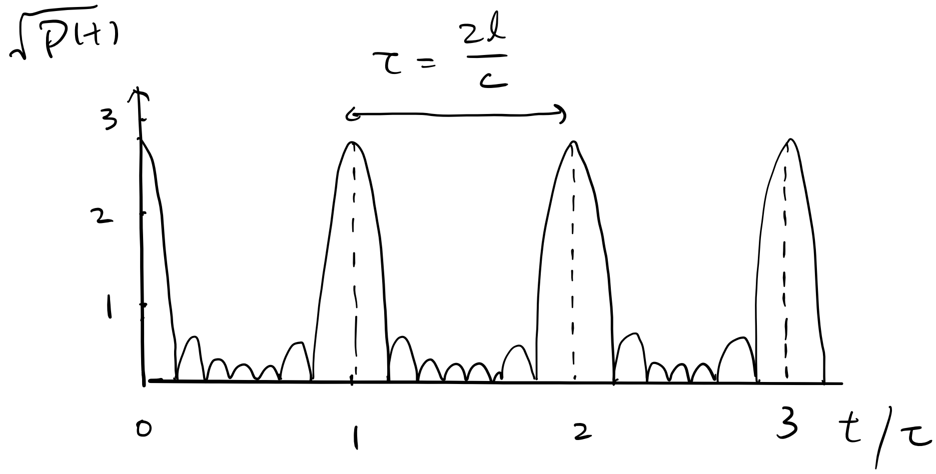
$$= \underbrace{E(t)\cdot \exp\left(i\,\frac{2\pi\omega_0}{\Omega}\right) = E(t)}_{\text{Periodic function.}}$$

Let $\phi_m = 0$, $C_m = \frac{1}{\sqrt{N}}$ $\longrightarrow$ # of locked osillating modes

$$E(t) = \frac{1}{\sqrt{N}} \cdot \sum_{m=1}^{N} e^{i(\omega_0 + m\Omega)t} = \frac{1}{\sqrt{N}} e^{i[(\omega_0 + (N+1)\Omega/2]t} \cdot \frac{\sin(N\Omega t/2)}{\sin(\Omega t/2)}$$

$$\boxed{P(t) \propto E^*(t)\,E(t) = \frac{1}{N}\,\frac{\sin^2(N\Omega t/2)}{\sin^2(\Omega t/2)}}$$

Plot of field amplitude: $\sqrt{p(t)}$




$(N = 8)$

## Comments:

1. Power is emitted in the form of pulse train;
periodicity $\tau = \frac{2\pi}{\Omega} = \frac{2l}{c}$. (round-trip transit time in cavity)

2. Peak power $P(s\tau)$, $(s = 0, 1, 2, 3 \ldots)$ = N times the average power, $N \equiv$ # of locked osillating mode.

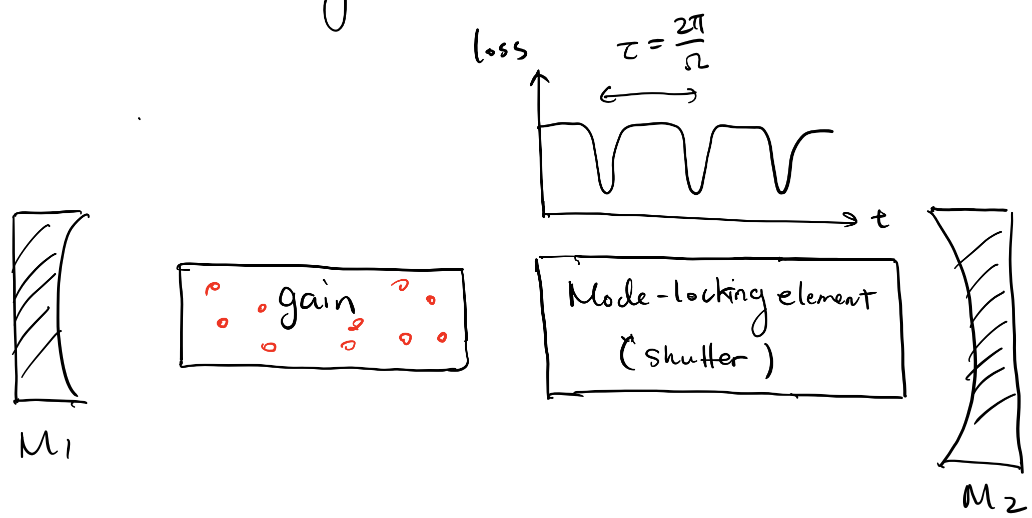
3. Pulse width (FWHM of the main peaks of $P(t)$)

$$\tau_0 = \frac{\tau}{N} \simeq \frac{\tau}{\Delta\omega/\Omega} = \frac{2\pi/\Omega}{\Delta\omega/\Omega} = \frac{2\pi}{\Delta\omega}$$

gain bandwidth ($2\pi \cdot \Delta\nu$)

$$\boxed{\text{Pulse width} \sim 1/\text{gain bandwidth}}$$

## 2. Mode-locking techniques




Basic idea:

① Insert a mode-locking element (shutter) inside a laser cavity

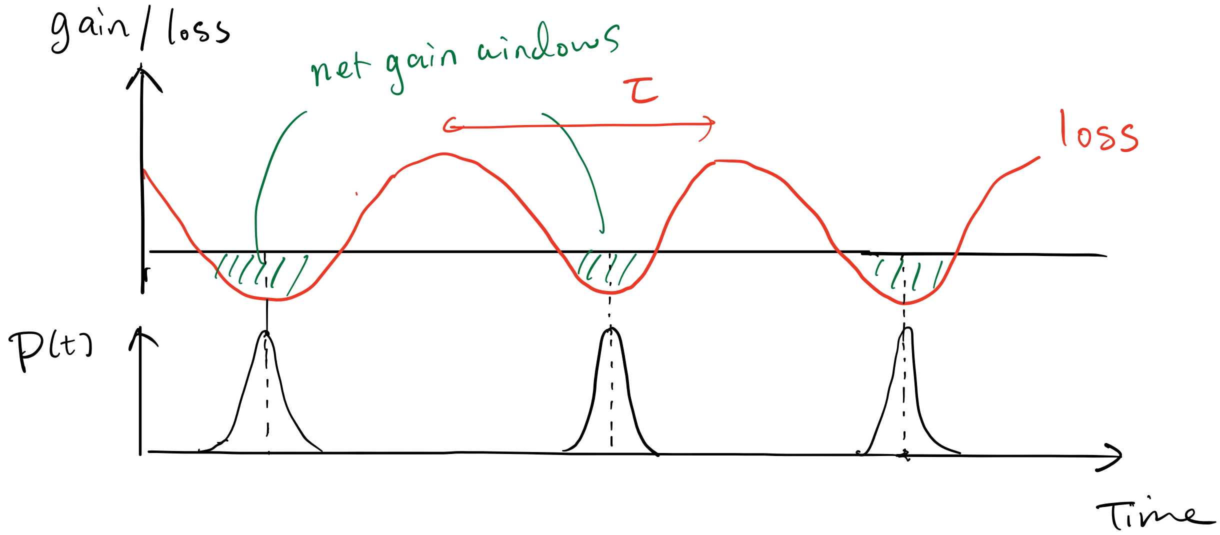
② The shutter "opens" and "closes" with a time period of $\tau$
$(\tau = \frac{2\pi}{\Omega})$

③ The shutter favors the transmission of short pulses, while suppressing the transmission of long pulses or CW signal.

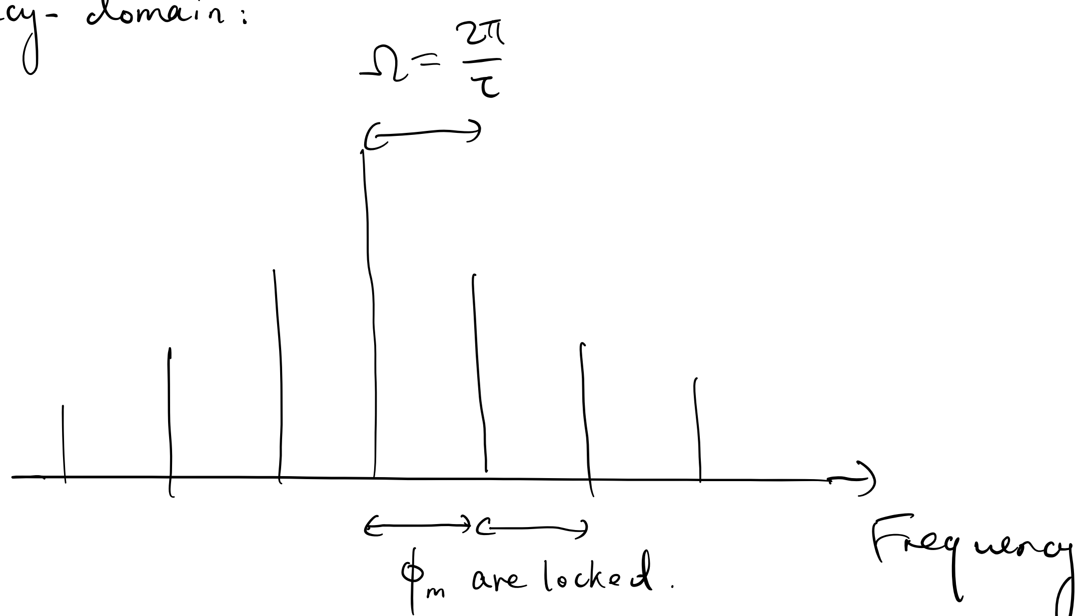
④ In steady state, the mode-locked laser generates ultrashort light pulses, with a rep-rate of $\frac{\Omega}{2\pi}$.

# Physical picture of mode-locking:

Time-domain:

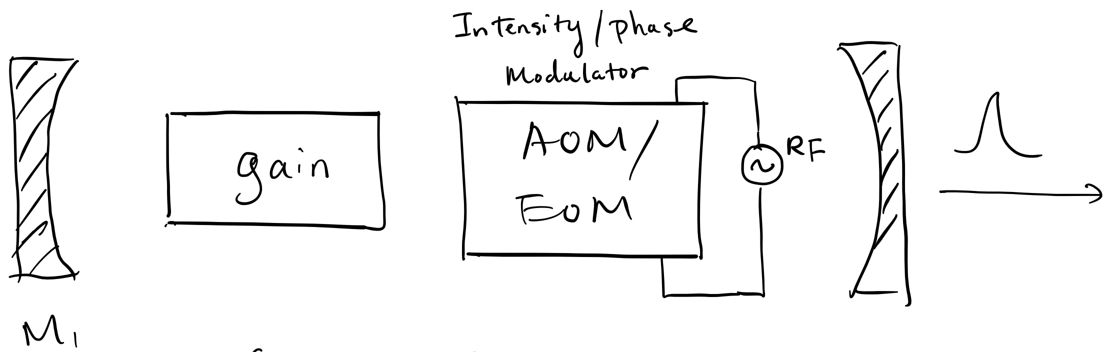



Frequency-domain:




Periodic modulation "locks" the adjacent modes.

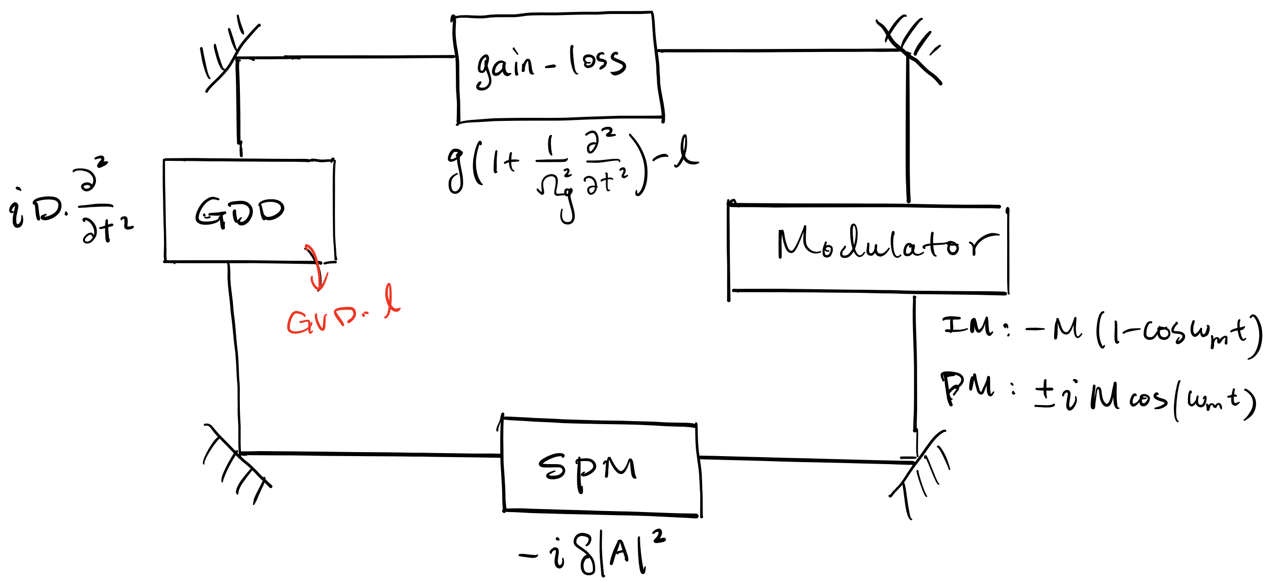
## Active mode-locking:




$$\omega_{RF} = \Omega$$

Q: How to calculate the steady-state pulse width?

A: Haus-Master Equation (HME). Ref. P244. Keller.




(Generic case in a MLL cavity)

In Steady-state:

slow time, fast time

$$T_R \cdot \frac{\partial A(T, t)}{\partial T} = \sum_i \Delta A_i = 0$$

changes in the pulse envelop due to gain, loss, modulation, dispersion, nonlinearity ...

Ignore SPM and GDD, HME of Actively MLL:

gain dispersion

IM

$$T_R \frac{\partial A(t,T)}{\partial T} = \left[ g\left(1 + \frac{1}{\Omega_g^2} \frac{\partial^2}{\partial t^2}\right) - l - M \cdot (1 - \cos \omega_m t) \right] A(t, T) = 0 \quad (1)$$

Assume $\frac{2\pi}{\omega_m} \gg$ pulse width.

$$M(1 - \cos \omega_m t) \simeq M \frac{\omega_m^2 t^2}{2}$$

Solution of 1 is a Gaussian pulse. (Ansatz)

$$A(t) = A_0 \exp\left(-\frac{t^2}{2\tau^2}\right)$$

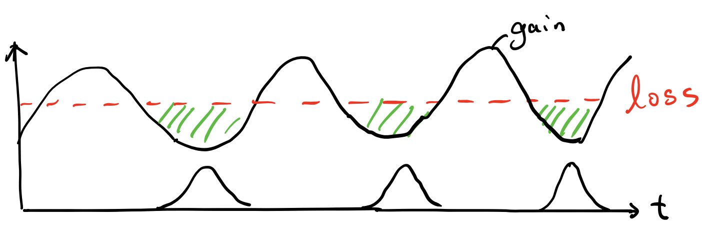
where $\tau = \sqrt[4]{\dfrac{\overbrace{2g}^{\text{Saturated gain}}}{M\omega_m^2 \underbrace{\Omega_g^2}_{\text{gain BW}}}}$

Pulse width : $\tau_p = 1.665 \cdot \tau = 0.446 \sqrt[4]{\dfrac{g}{M}} \cdot \sqrt{\dfrac{1}{f_m \Delta f_g}}$

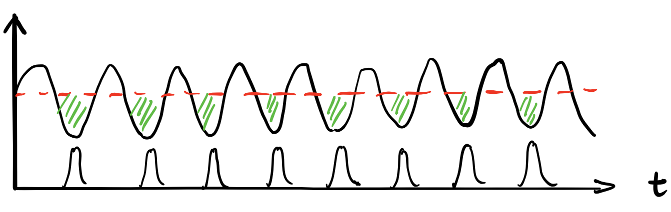
## Comments :

1 Pulse width depends more on modulation freq. ($f_m$) and gain bandwidth.

Small $f_m$ :



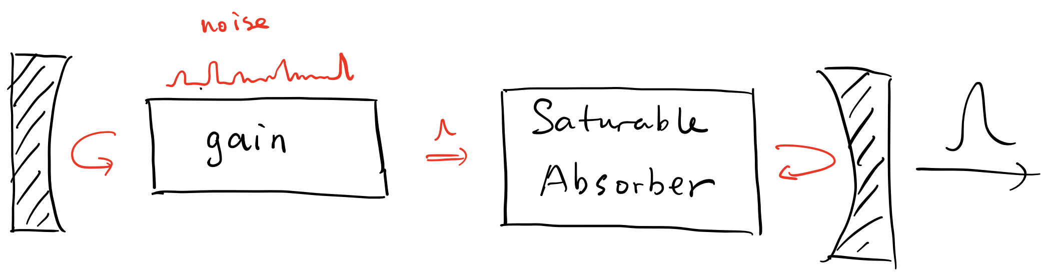

Large $f_m$ :




reason: narrower net gain window !

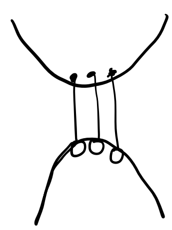
② Increasing Modulation strength or reducing the loss ($g=\ell$) <u>does not</u> significantly reduce $\tau_p$

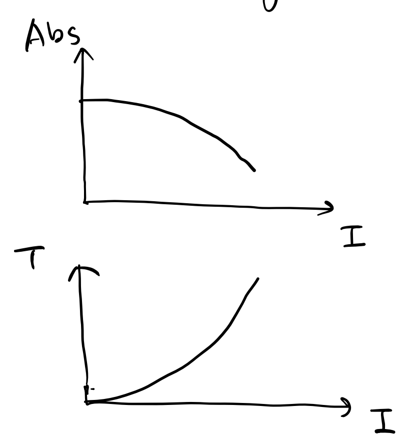
## Passive mode-locking.

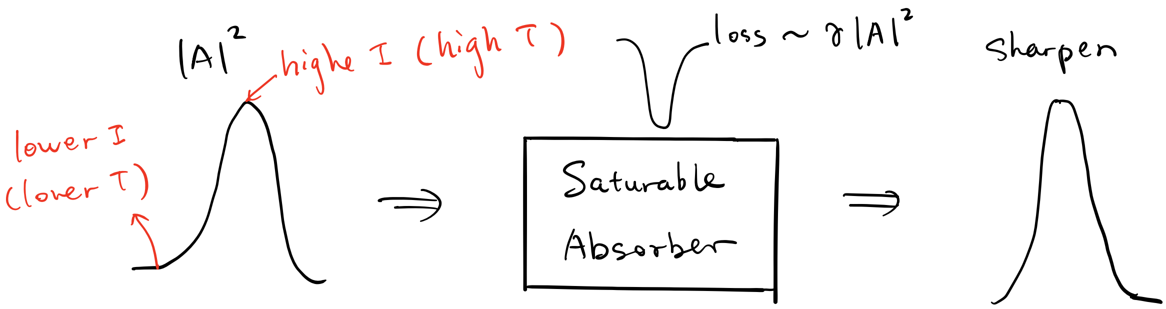



<u>Saturable absorber</u>: A device whose T ↑ as intensity ↑

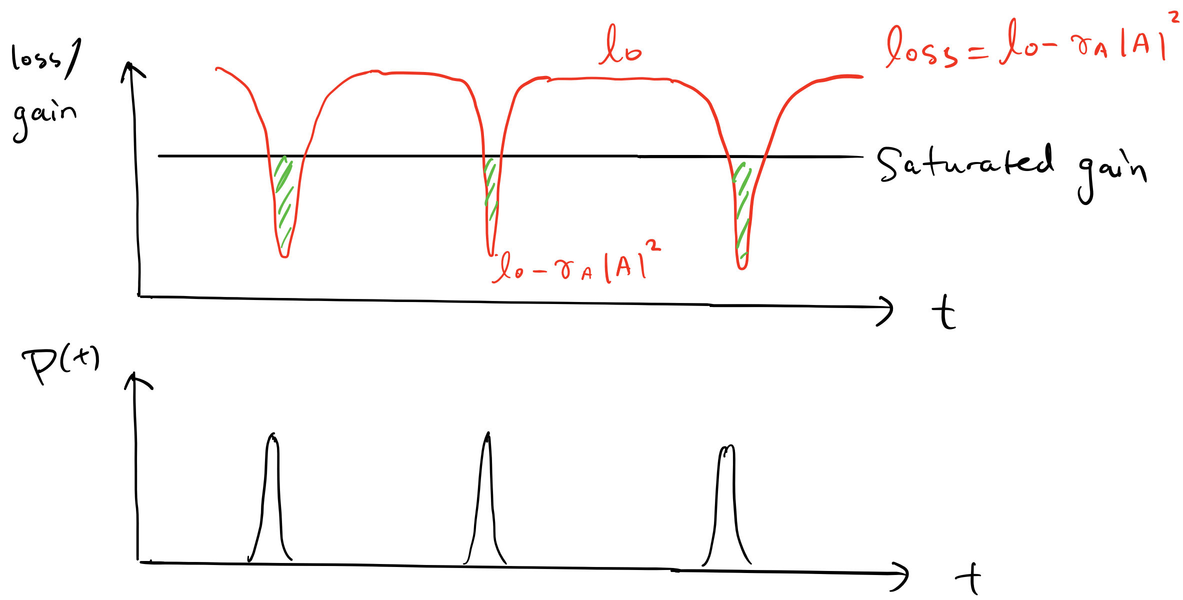
e.g. Semiconductor
graphene, CNT.








Physical picture:



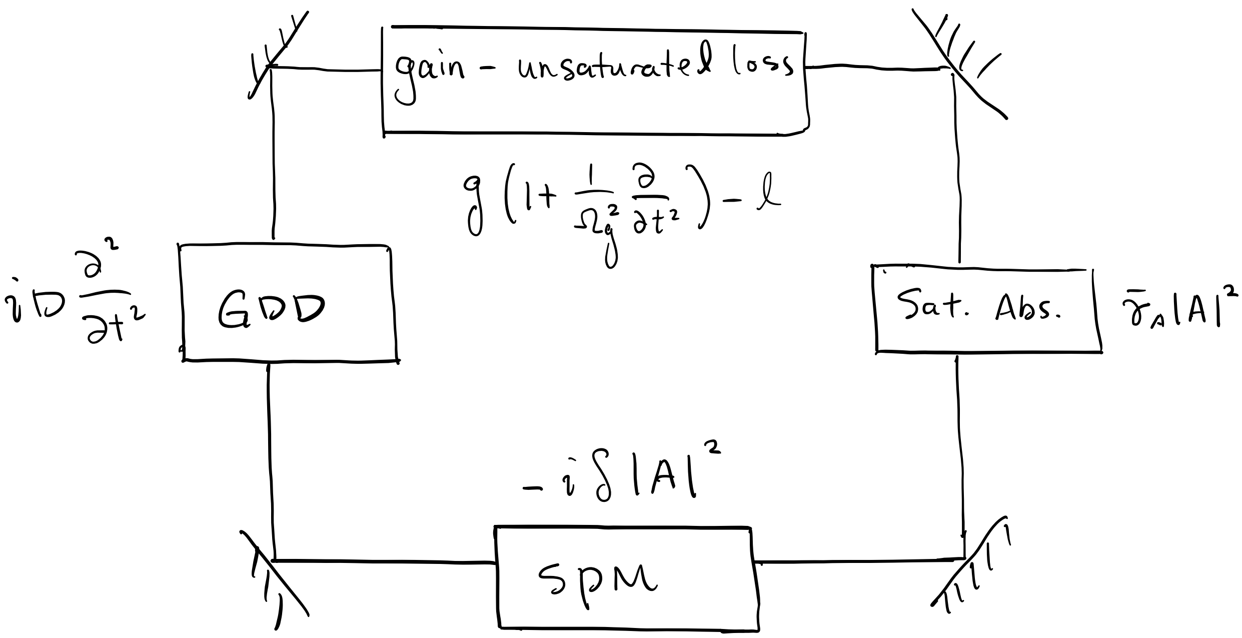

## Comments:

① Sat. Abs. can be regarded as a "fast shutter", in which the loss modulation is induced by laser itself.

② Sat. Abs. does not require external RF-driving — Passive

③ Since the "net gain time window" is small, Passive mode-locking generally produces shorter pulses!

HME in generic case: (P446. U. Keller)




Total loss:

$$l(t) = l_0 - \bar{\gamma}_A |A(t)|^2$$

Ignore the SPM and GDD, HME:

$$T_R \cdot \frac{\partial A}{\partial T} = g\left(1 + \frac{1}{\Omega_g^2}\frac{\partial^2}{\partial t^2}\right)A - \left(l_0 - \bar{\gamma}_A |A|^2\right)A = 0$$

Solution: $A(t) = A_0 \operatorname{sech}\left(\frac{t}{\tau}\right)$

where $\tau = \sqrt{\frac{2g}{\Omega_g^2 \bar{\gamma}_A |A_0|^2}} = \frac{4D_g}{\bar{\gamma}_A E_p}$

pulse energy.

$E_p = \int P(t)\,dt = \int |A|^2 dt$

$= A_0^2 \int \operatorname{sech}^2\left(\frac{t}{\tau}\right) dt$

$= 2A_0^2 \tau$

## 3. Comparison between active & passive mode-locking.

| | Active | Passive |
|---|---|---|
| Shape: | $A(t) = A_0 \exp\left(-\frac{t^2}{2\tau^2}\right)$ | $A(t) = A_0 \operatorname{sech}\left(\frac{t}{\tau}\right)$ |
| width | $\tau^4 = \frac{2D_g}{M\omega_m^2}$ | $\tau^2 = \frac{2D_g}{\bar{\gamma}_A |A_0|^2}$ |

$\Downarrow$

$$\boxed{M\omega_m^2 \Longleftrightarrow \frac{\bar{\gamma}_A |A_0|^2}{\tau^2}}$$

## Comments

Passive mode-locking with a saturable absorber is similar to mode-locking with a loss modulator, the loss modulation is induced by pulse itself. (A.K.A. Self-amplitude modulation, SAM)