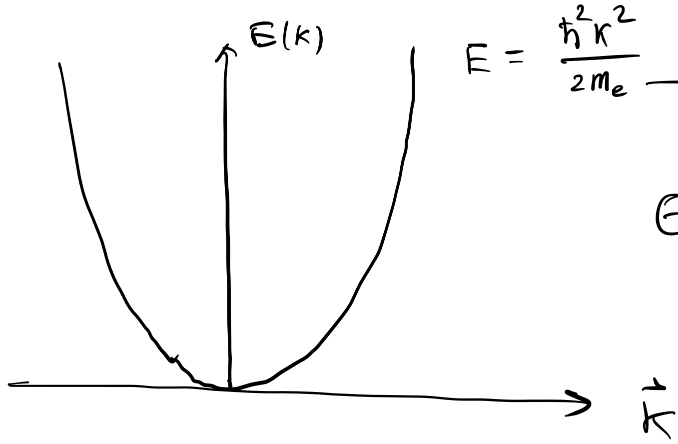
Lecture 17. Semiconductor Lasers I

Learning objectives:

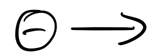
1. Review of semiconductor physics.
   - Density of state
   - Fermi-Dirac distribution
   - Fermi / Quasi-Fermi level
2. Gain and absorption in semiconductors
3. Typical semiconductor lasers

1. Review of semiconductor physics. (Yariv. P673)

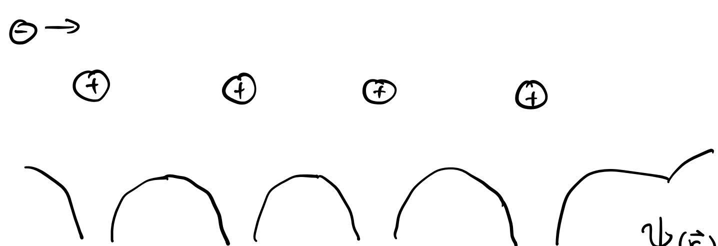
① Free-electron:



$$E = \frac{\hbar^2 k^2}{2m_e}$$ — mass of free electrons



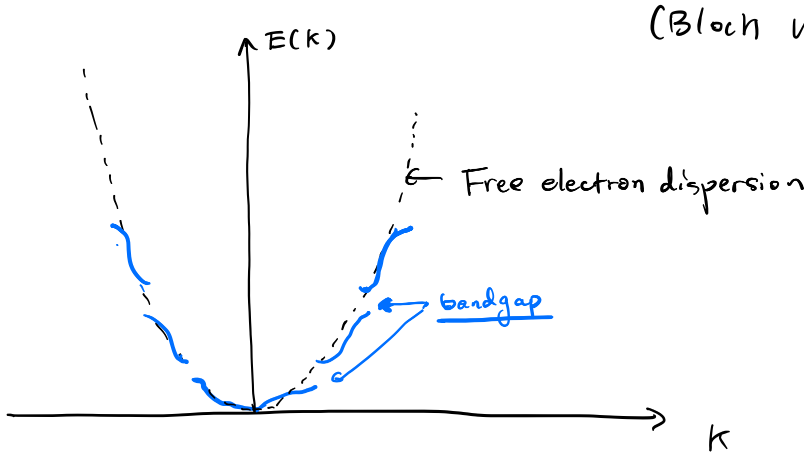
② Electrons in real crystals:



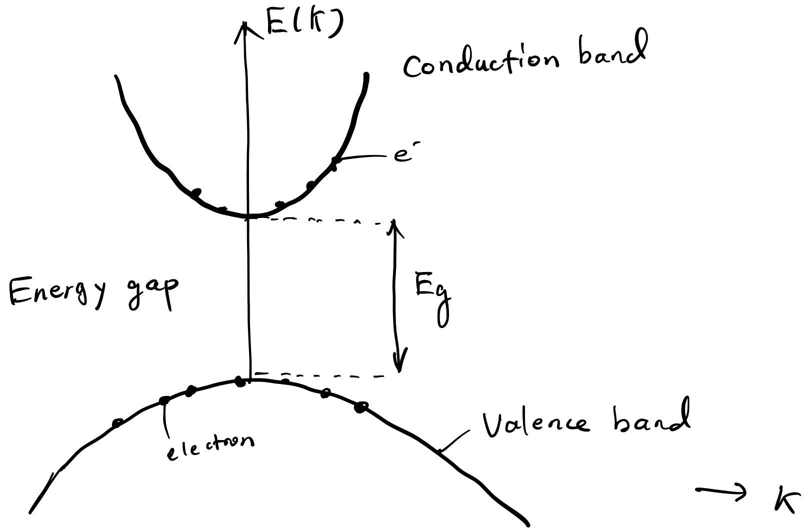
periodic potential $u(r)$

$\psi(\vec{r}) = u_k(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}$

(Bloch wave)



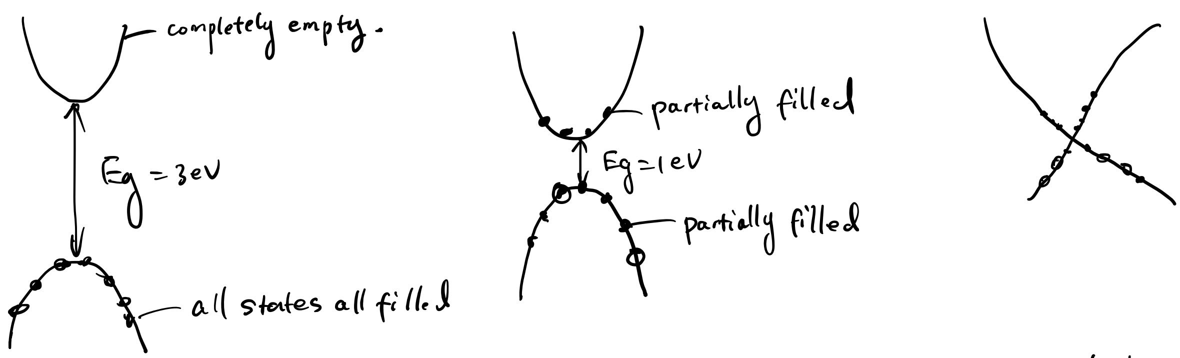
Let's zoom-in the bandgap region:



Qualitative picture:

1. All materials with crystal lattice has bandgaps.

2. At room temperature, thermal energy ($k_B T = 0.026\,eV$)



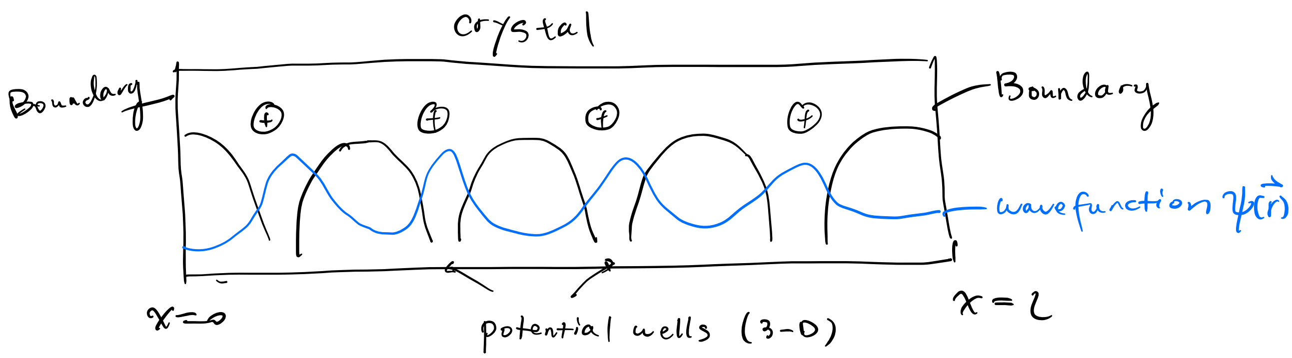
insulator   Semiconductor   metal.

Next question: how to calculate the # of electrons?

Answer:

$$\underbrace{n}_{\text{\# of eleltron per unit volume}} = \int \underbrace{\rho(E)}_{\text{Density of state (DOS)}} \; \underbrace{P(E)}_{\text{Probability of State is occupied (Fermi-Dirac distribution)}} \cdot dE$$

① Density of states (DOS)



Boundary condition requires:

$\psi(0) = \psi(L) = 0.$ $\quad$ ($e^{ikL} = 0$ at $L=0$ and L)

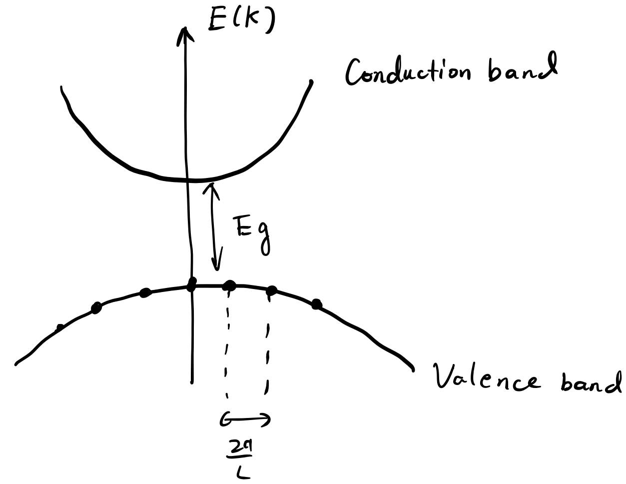
$\Rightarrow \quad k = m \cdot \frac{2\pi}{L}, \quad m = 1.2.3.$

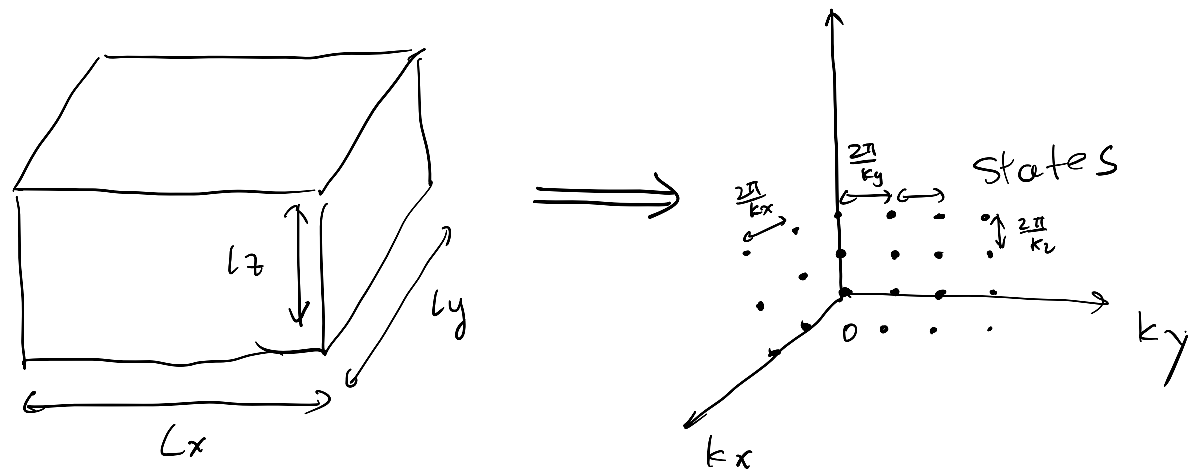
$\Rightarrow$ x direction: $\quad k_x = l \cdot \frac{2\pi}{L}. \quad l = 1.2.3 \ldots$

y 〃 $\quad k_y = m \frac{2\pi}{L} \quad m = 1.2.3 \ldots$

z 〃 $\quad k_z = n \frac{2\pi}{L} \quad n = 1.2.3 \ldots$

Physical meaning:





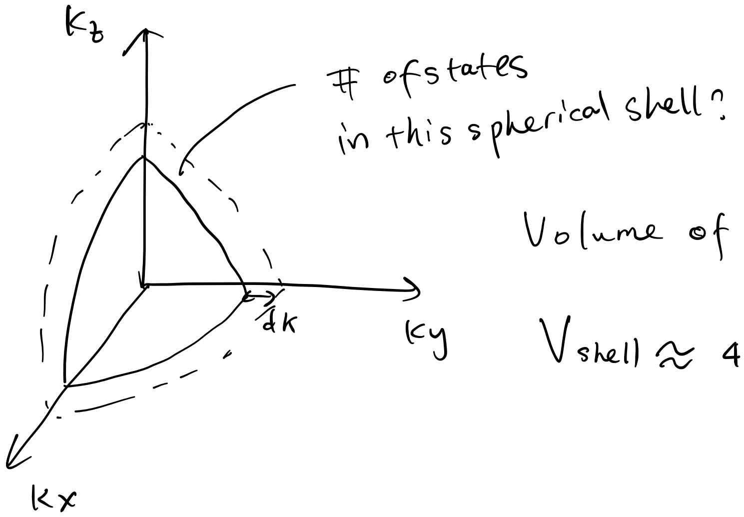
real space

k- space.
(reciprocal space)

In k-space: each cell has a volume of

$$V_k = \Delta k_x \cdot \Delta k_y \cdot \Delta k_z = \left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right) = \frac{8\pi^3}{V}$$

Volume of crystal



Volume of sphererical shell:

$$V_{shell} \approx 4\pi k^2 dk$$

So, the number of states in the spherical shell:

$$\rho(k)\cdot dk = 2\cdot\frac{V_{shell}}{V_k} = 2\cdot\frac{4\pi k^2 dk}{\frac{8\pi^3}{V}} = \frac{k^2 V}{\pi^2}dk.$$

number of states per unit volume of k-space.

each state can accomodate two electrons (i.e. spin up. spin down)

We know that:

$$E_c(k) = \frac{\hbar^2 k^2}{2m_c} \Rightarrow k = \sqrt{\frac{2m_c E}{\hbar}}$$

$$E_v(k) = \frac{\hbar^2 k^2}{2m_v} \Rightarrow k = \sqrt{\frac{2m_v E}{\hbar}}$$

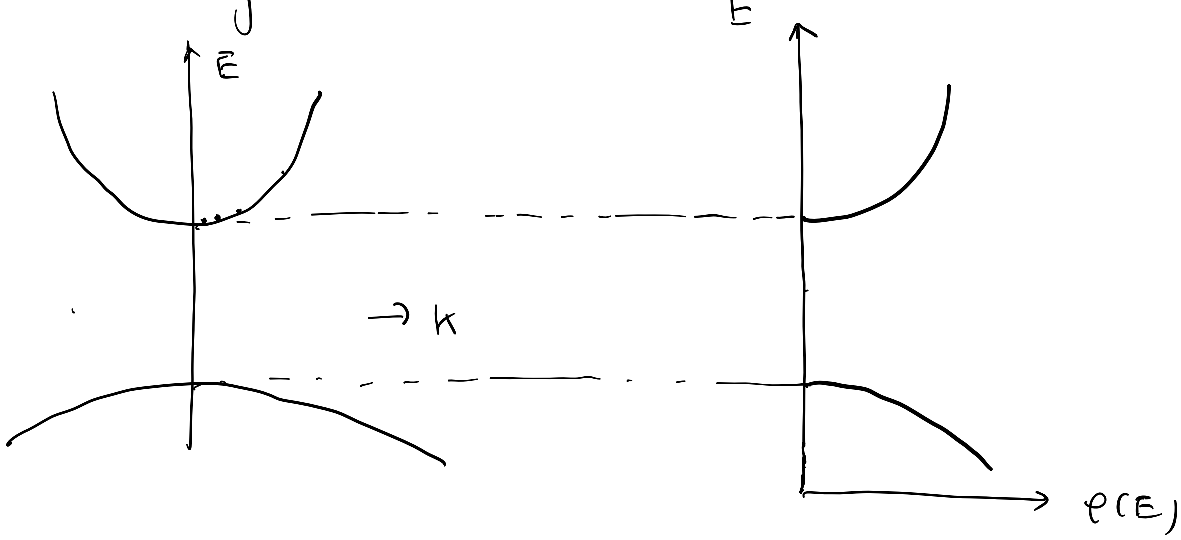
So $dk = \left(\frac{2mE}{\hbar^2}\right)^{-\frac{1}{2}} \cdot \frac{m}{\hbar^2} dE$

So density of state $\rho(E)$ (the number of electronic states per unit energy interval, per unit crystal volume)

$$\rho(E)\,dE = \frac{1}{V}\rho(k)\cdot dk$$

$$= \frac{k^2 dk}{\pi^2} = \frac{2mE}{\pi^2\hbar^2}\left(\frac{2mE}{\hbar^2}\right)^{-\frac{1}{2}}\cdot\frac{m}{\hbar^2}dE = \boxed{\frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} dE}$$

Band diagram



Comments:

① E ↑, ρ(E) ↑. Band is "larger" in k space
Can accommodate more states.

② No states in the bandgap!

② The Fermi-Dirac distribution

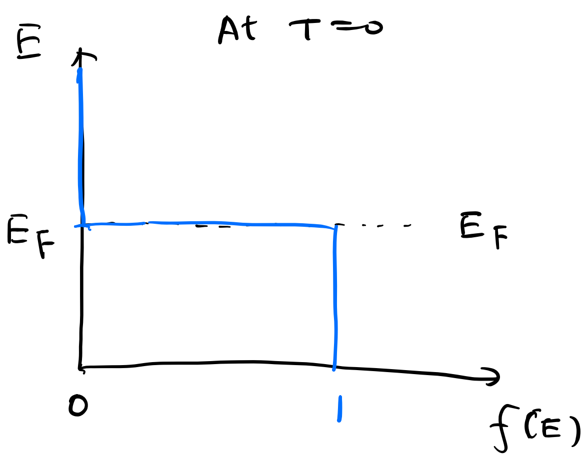
Fermi-Dirac distribution:

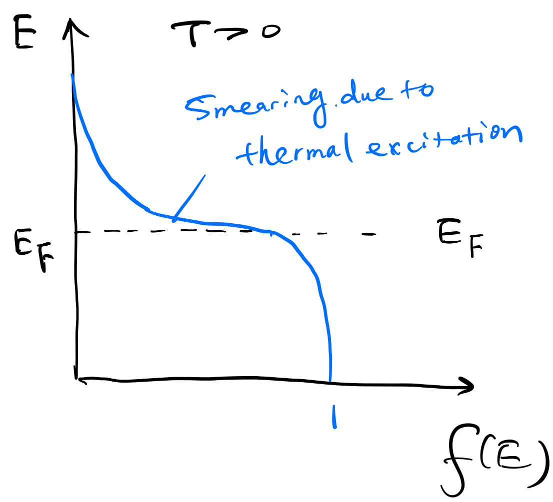
$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{k_B T}\right)} + 1}$$

Fermi level.

Boltzman constant: 1.381×10^{-23} J/K

Probability that an electron state at energy E is occupied by an electron.





Comments:

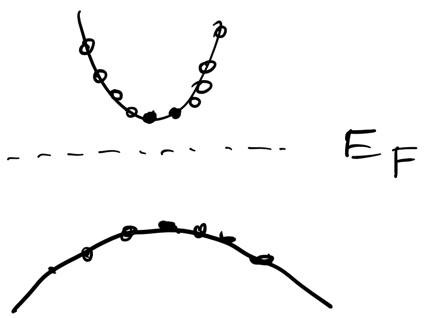
① At $T=0$, Below E_F, all states are occupied ($f(E)=1$)
Above E_F, all states are empty ($f(E)=0$)

② $T>0$, Below E_F, most states are occupied
Above E_F, most states are empty.

Note, $f(E)$ is the Fermi-Dirac distribution for electrons

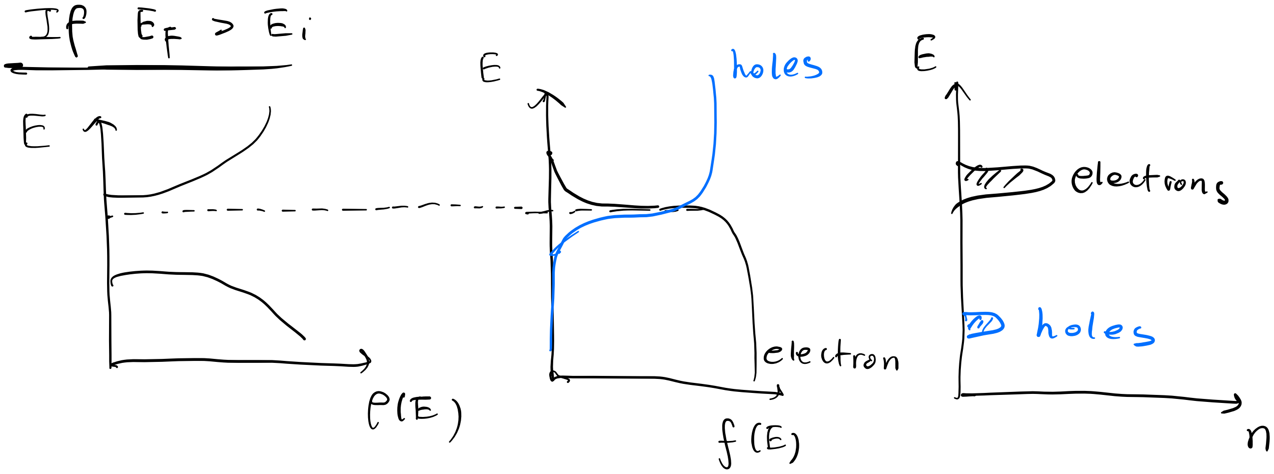
For holes, $1 - f(E)$

Let's first consider thermal equilibrium:

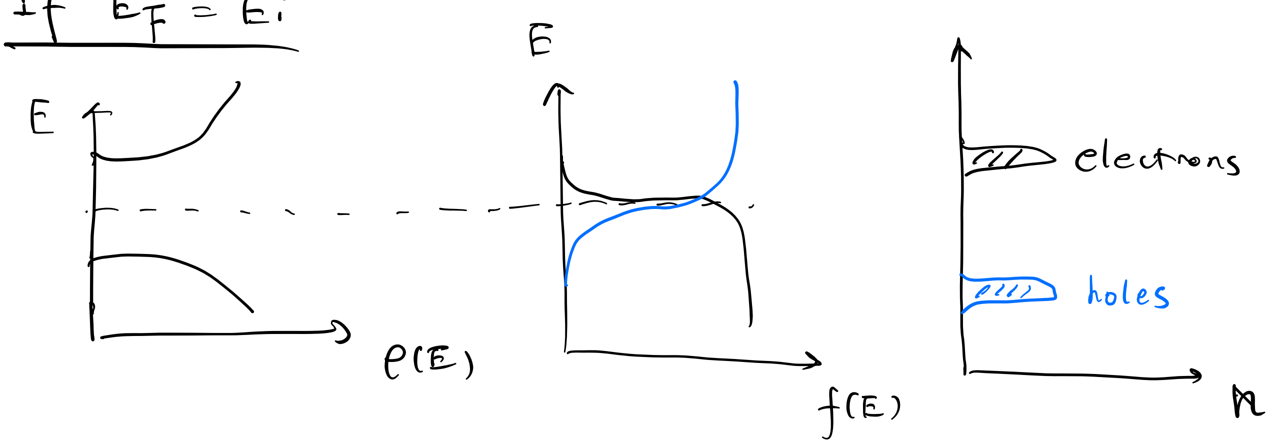


$$n = \int_0^\infty \rho(E) f(E)\, dE$$

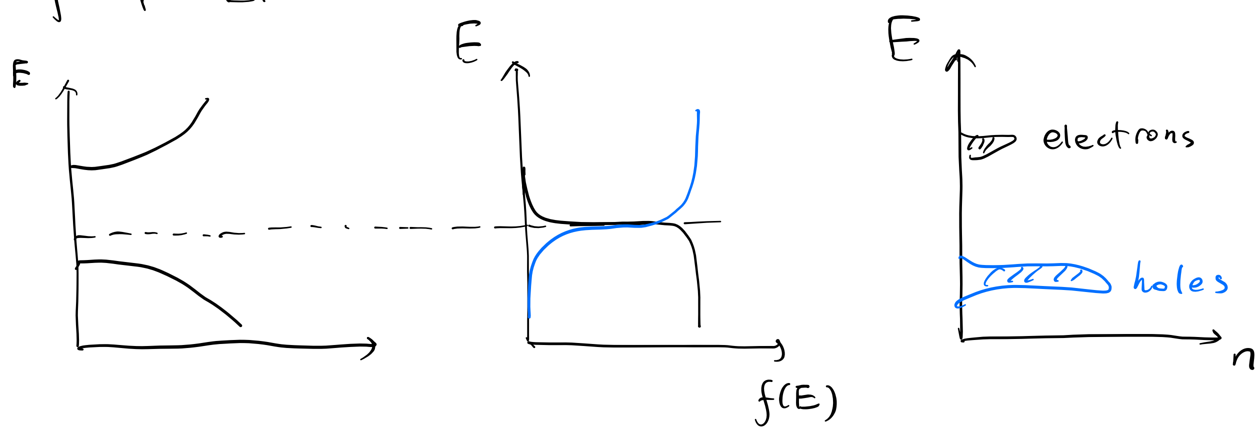
If $E_F > E_i$



If $E_F = E_i$



If $E_F < E_i$



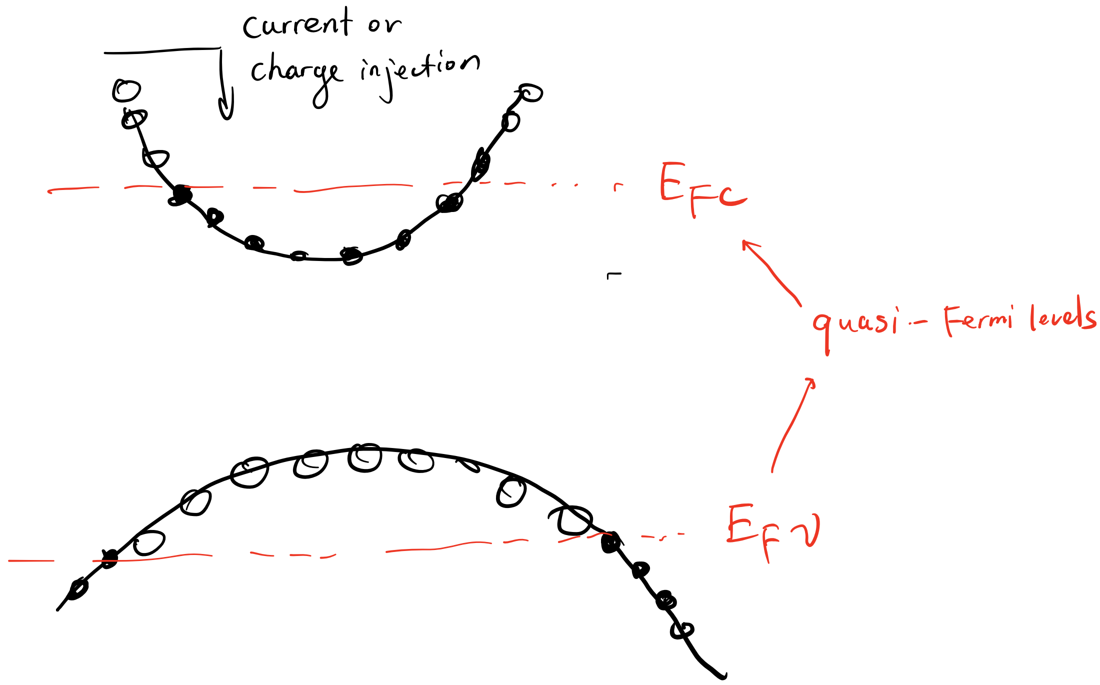
Summary:

① When $E_F > E_i$. # of electrons > # of holes.
(n-type)

② When $E_F = E_i$ # of electrons = # of holes
(intrinsic)

③ When $E_F < E_i$ # of electrons < # of holes
(p-type)

Non-thermal equilibrium (i.e. charge injection etc)

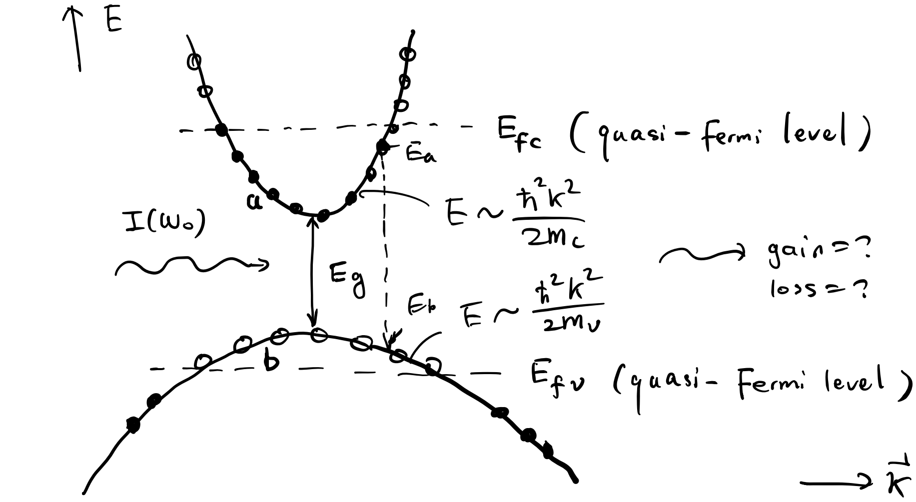


We use "quasi-Fermi levels" (E_{Fc}, E_{Fv}) to describe electron states occupation probabilities when semiconductor is out of thermal equilibrium.

2. Gain and absorption in semiconductor media (P680, Yariv)

Problem to study:

Non-equilibrium (e.g. current injection)



For any type of gain media (atomic systems, semiconductor)

Imaginary part of the electric susceptibility:

$$\chi''(\omega_0) = \frac{(N_1 - N_2)\lambda_0^3}{8\pi^3 t_{spon}\,\Delta\nu\, n} \frac{1}{1 + 4(\nu-\nu_0)^2/(\Delta\nu)^2} \qquad (P228.\ \text{Yariv})$$

gain coefficient:

$$\gamma(\omega_0) = -\frac{k}{n^2}\chi''(\omega_0) \qquad (P229.\ \text{Yariv})$$

$$= \frac{(N_2 - N_1)\lambda_0^2}{4\pi^2 t_{spon}} \cdot \frac{T_2}{\pi[1 + (\omega-\omega_0)^2 T_2^2]}$$

where $T_2 = \frac{1}{\pi\Delta\nu}$, is the mean life time for coherent interaction of electrons with monochromatic field. $T_2 \sim 10^{-12}$ s for semiconductors.

Downward transition rate from a to b:

$$R_{a\to b} \propto f_c(E_a)\left[1 - f_v(E_b)\right]$$

Upward transition rate from b to a.

$$R_{b\to a} \propto f_v(E_b)\left[1 - f_c(E_a)\right]$$

So population inversion:

$$N_2 - N_1 = \boxed{\frac{\rho(k)\,dk}{V}}\left[R_{a\to b} - R_{b\to a}\right]$$

$$= \frac{\rho(k)\,dk}{V}\left\{f_c(E_a)\left[1 - f_v(E_b)\right] - f_v(E_b)\left[1 - f_c(E_a)\right]\right\}$$

$$= \frac{\rho(k)\,dk}{V}\underbrace{\left[f_c(E_a) - f_v(E_b)\right]}_{\text{Fermi-dirac distribution}}$$

Note:

in conventional lasers, Boltzman distributions

in semiconductor lasers: Fermi dirac distributions.

$$d\gamma(\omega_0) = \frac{\rho(k)\,dk}{V}(f_c - f_v)\frac{\lambda_0^2}{4n^2\tau}\left(\frac{T_2}{\pi\left[1 + (\omega-\omega_0)^2 T_2^2\right]}\right)$$

$$\gamma(\omega_0) = \int_0^\infty \frac{\rho(k)}{V}(f_c - f_v)\frac{\lambda_0^2}{4n^2\tau}\left(\frac{T_2}{\pi\left[1 + (\omega-\omega_0)^2 T_2^2\right]}\right) \quad (1)$$

how does k depend on ω?

Transition energy:

$$\hbar\omega = E_g + \frac{\hbar^2 k^2}{2m_c} + \frac{\hbar^2 k^2}{2m_v} = E_g + \frac{\hbar^2 k^2}{2m_r}$$

$$\frac{1}{m_r} = \frac{1}{m_c} + \frac{1}{m_v}$$

From the above equation, we get

$$\begin{cases} d\omega = \dfrac{\hbar}{m_r} k \, dk \\ k = (\hbar\omega - E_g)^{1/2} \cdot \left(\dfrac{2m_r}{\hbar^2}\right)^{1/2} \end{cases}$$

Plug in ①.

$$\gamma(\omega_0) = \int_0^\infty (\hbar\omega - E_g)^{1/2} \left(\frac{2m_r}{\hbar^2}\right)^{1/2} \frac{m_r \lambda_0^2 [f_c(\omega) - f_v(\omega)]}{4n^2\pi^2 h\tau} \cdot \frac{T_2}{\pi[1+(\omega-\omega_0)^2 T_2^2]} d\omega$$

$$\simeq \frac{\lambda_0^2}{8\pi^2 n^2 \tau} \left(\frac{2m_c m_v}{\hbar(m_v + m_c)}\right)^{3/2} \left(\omega_0 - \frac{E_g}{\hbar}\right)^{1/2} [f_c(\omega_0) - f_v(\omega_0)]$$

where we used $\hbar\omega_0 = E_a - E_b$

To have gain $(\gamma(\omega_0) > 0)$, we must have $\underline{f_c(\omega_0) > f_v(\omega_0)}$

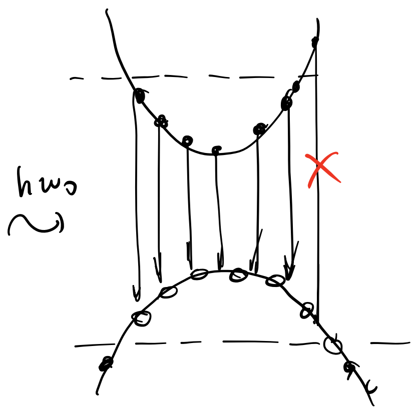
i.e. $$\frac{1}{e^{(E_a - E_{Fc})/k_B T} + 1} > \frac{1}{e^{(E_b - E_{Fv})/k_B T} + 1} \qquad ②$$

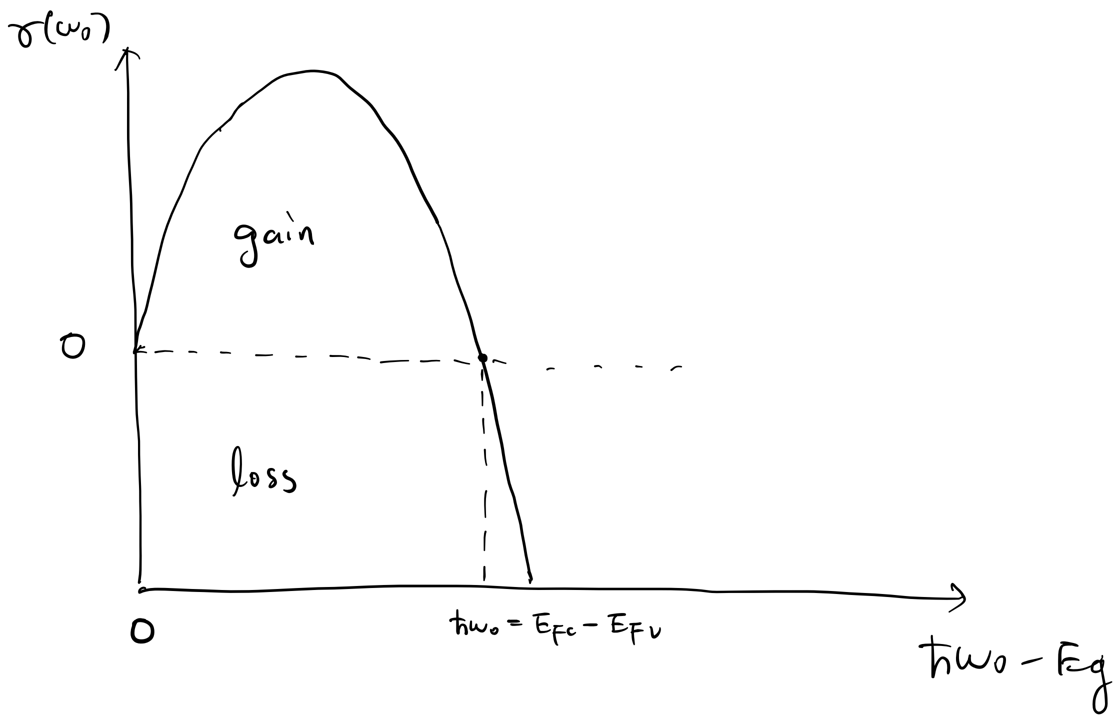
Recall that $E_a - E_b = \hbar\omega_0$.

② leads to:

$$\hbar\omega_0 < E_{Fc} - E_{Fv}$$

<u>Physical meaning</u>: only frequencies whose photon energies $\hbar\omega_0$ are smaller than the quasi-Fermi level seperations are amplified.





Comments:

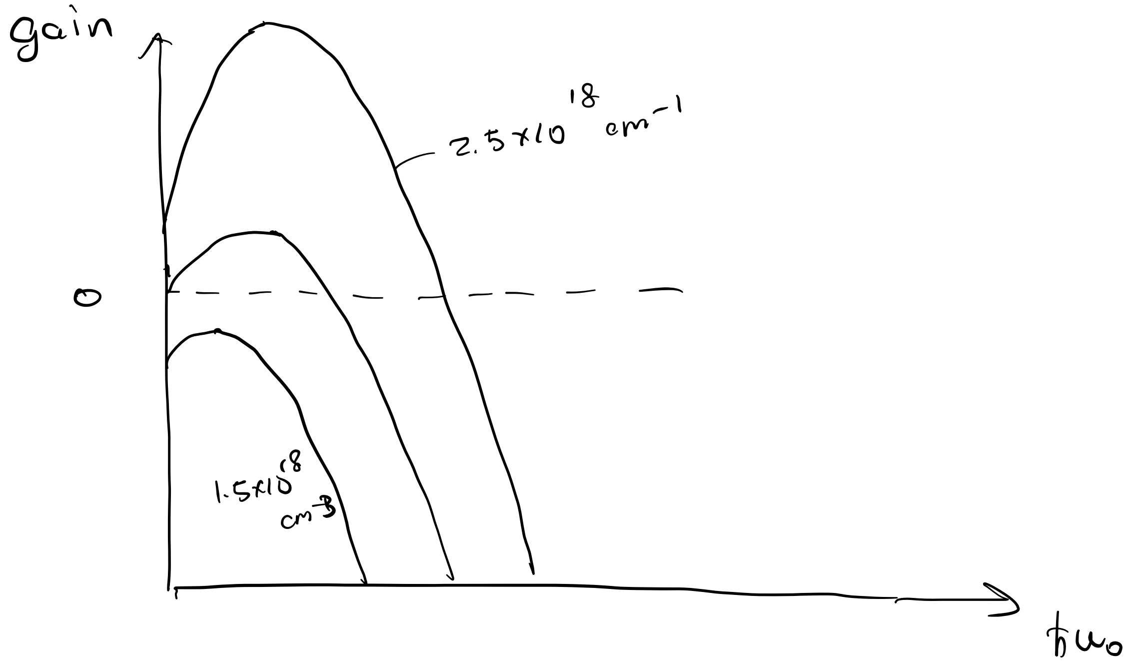
① when $\hbar\omega_0 < E_g$, no gain, no absorption (no transition)

② when $\hbar\omega_0$ is slightly greater than E_g, gain ↑

③ gain becomes zero again when $\hbar\omega_0 = E_{Fc} - E_{Fv}$.

④ when $\hbar\omega_0 > E_{Fc} - E_{Fv}$, loss ↑ due to transitions.

⑤ The spectral profile is determined by the Fermi-Dirac distribution.

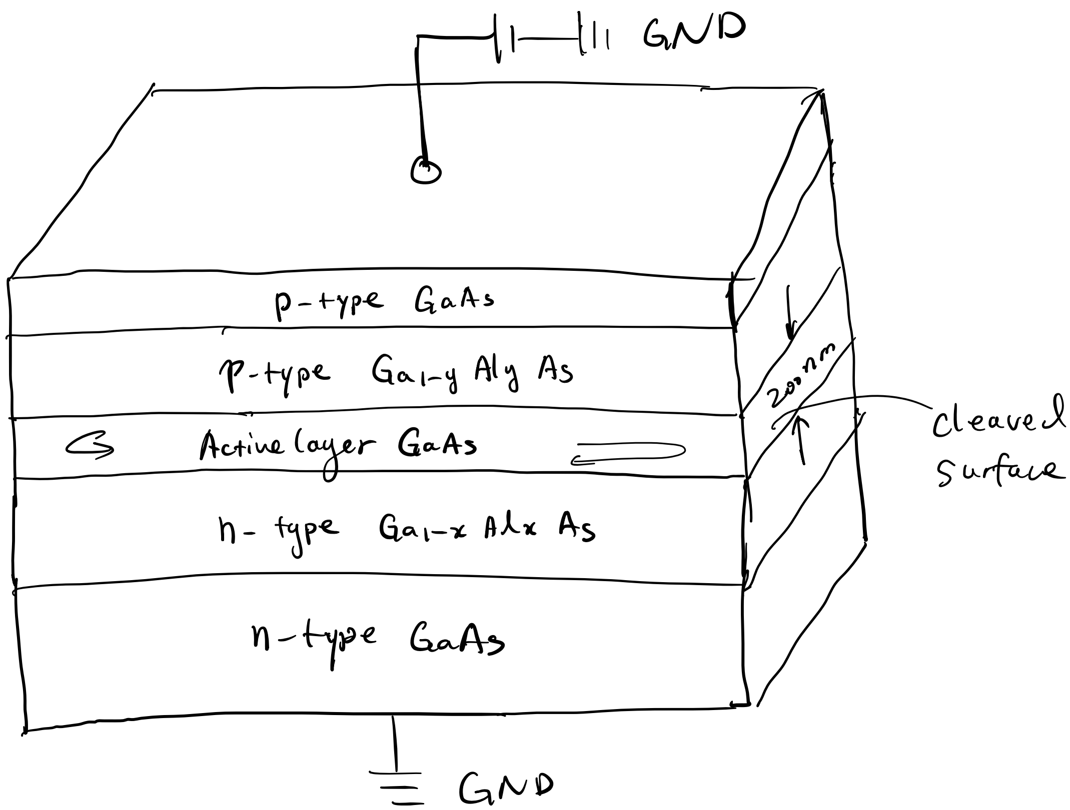


gain can be very high for semiconductor gain media!

4. Typical Semiconductor lasers

Double Heterostructure (DH) Laser (P686. Yariv)

e.g. $GaAs / Ga_{1-x} Al_x As$ lasers

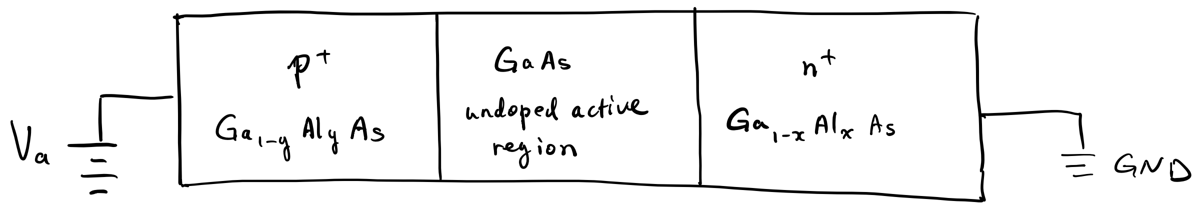


Active layer GaAs can serve as

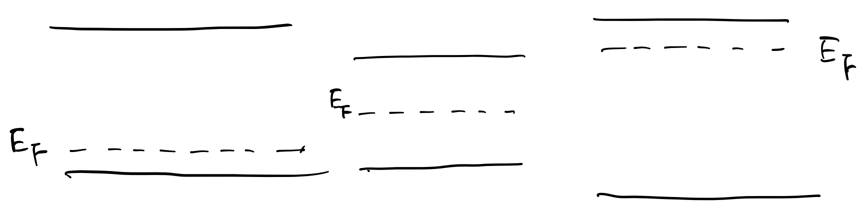
① lasing media

② wave guide.

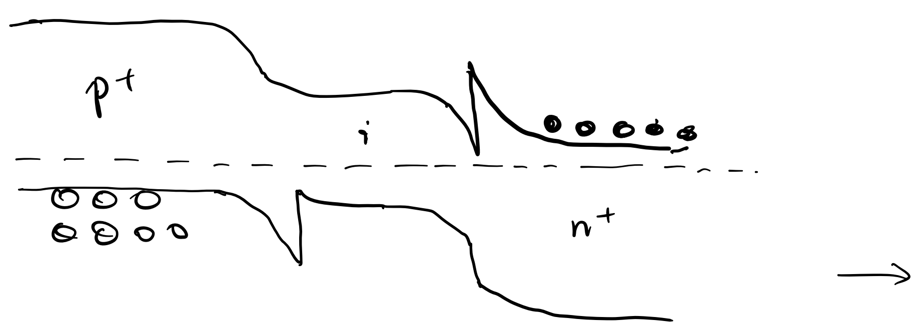
Cleaved surface can serve as reflectors



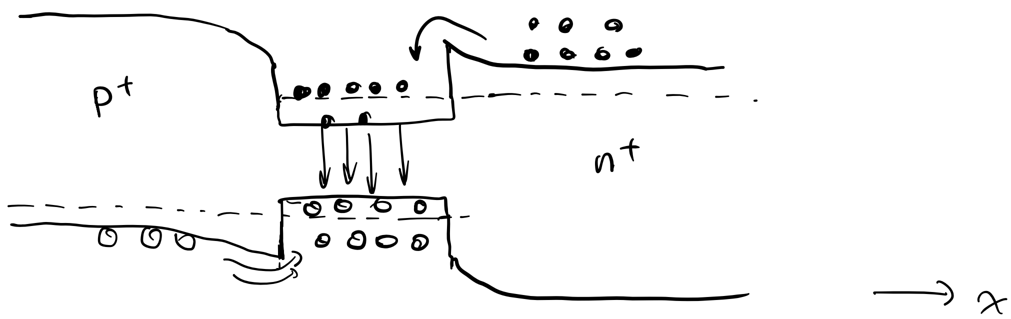
Before contact:



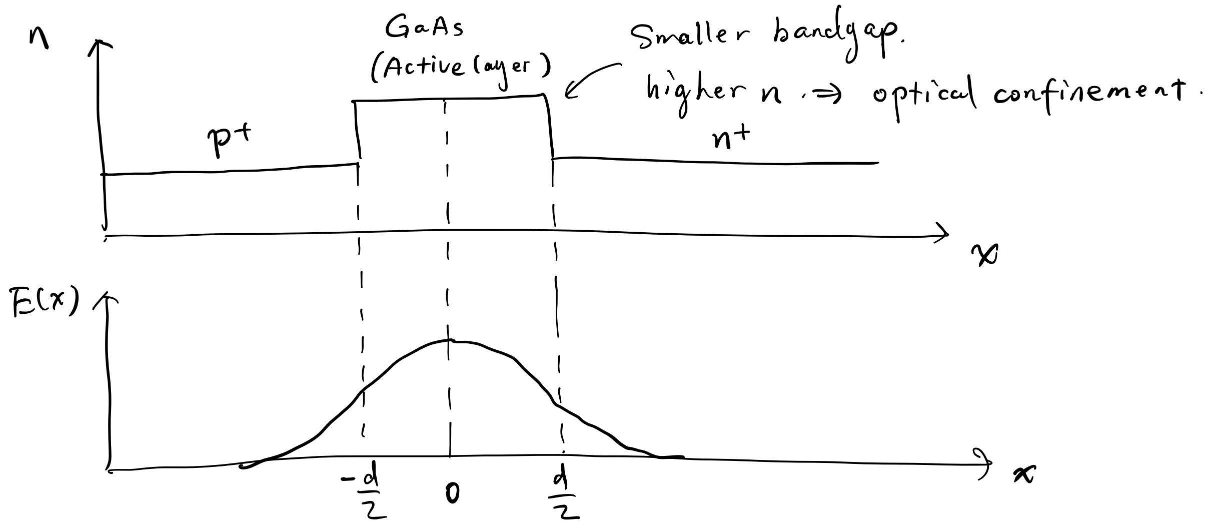
After contact (V=0)



Forward bias (V>0)



High concentration of both e^- and h^+ in the smaller-bandgap active region → carrier confinement.



Confinment factor: $\Gamma = \dfrac{\int_{-\frac{d}{2}}^{\frac{d}{2}} |E(x)|^2 dx}{\int_{-\infty}^{\infty} |E(x)|^2 dx}$