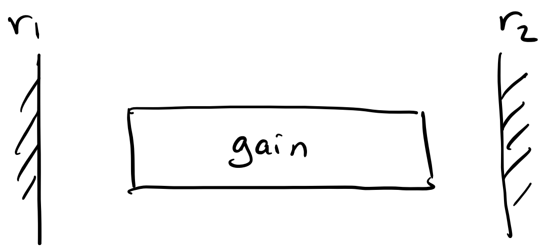
# Lecture 16 : Laser oscillation II

Learning objectives:

1. Optimum output coupling
2. Muti-mode lasing
3. Mode-locking
4. Methods for mode-locking. $\begin{cases} \text{Active} \\ \text{passive} \end{cases}$

## 1. Optimum output coupling (Yariv P249 ~ P251)



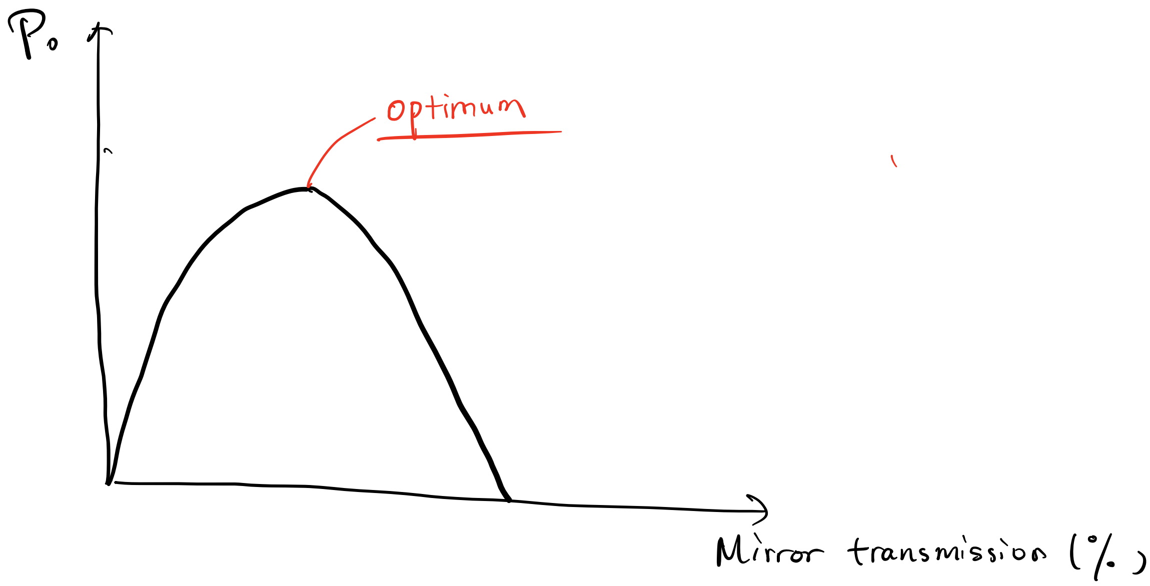

threshold condition: gain = loss.

threshold gain: $\gamma_t(\omega) = \alpha - \frac{1}{l}\ln r_1 r_2$.

Problem:

① If both $r_1$ and $r_2$ are unity,
$\gamma_t(\omega)$ and $P_{th}$ are minimized, but no output

② If $r_1$ and $r_2$ are very small,
$\gamma_t(\omega)$ and $P_{th}$ are super-large, cannot lase.




## Output power:

unsaturated gain ($\gamma$ is gain per unit length $\gamma \cdot l = g$)

$$P_o = I_s A T \left( \frac{g_o}{L} - 1 \right)$$

Saturation intensity

round-trip loss. $(1 - r_1 r_2 \exp(-2\alpha l))$

Let $L = L_i + T$ ← mirror transmission

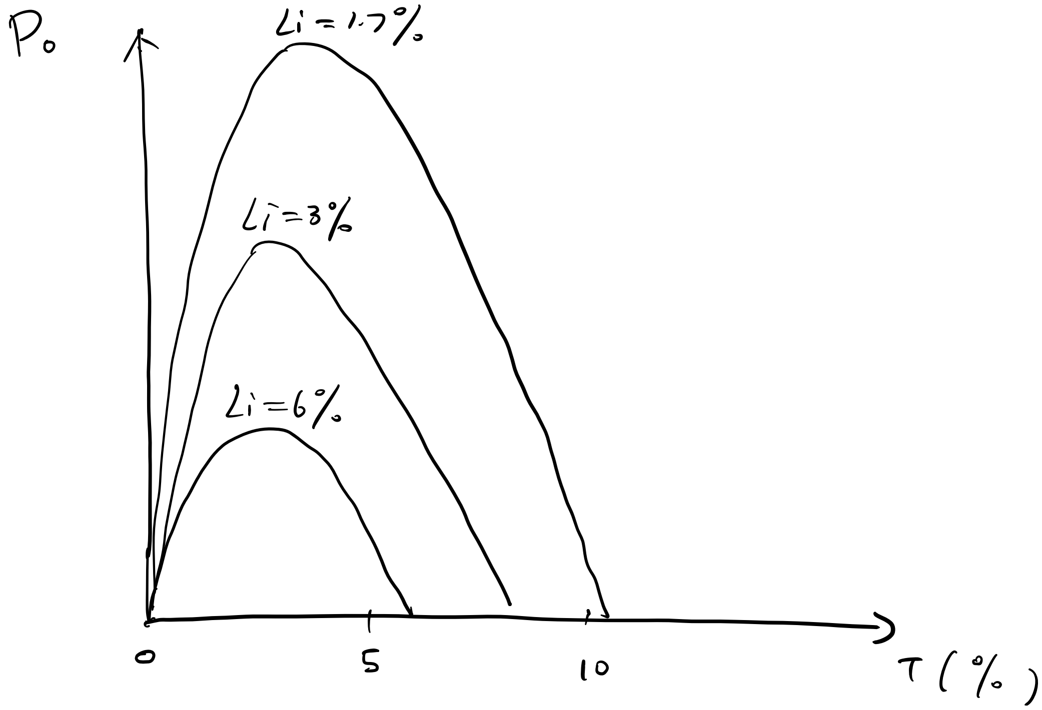
residual loss

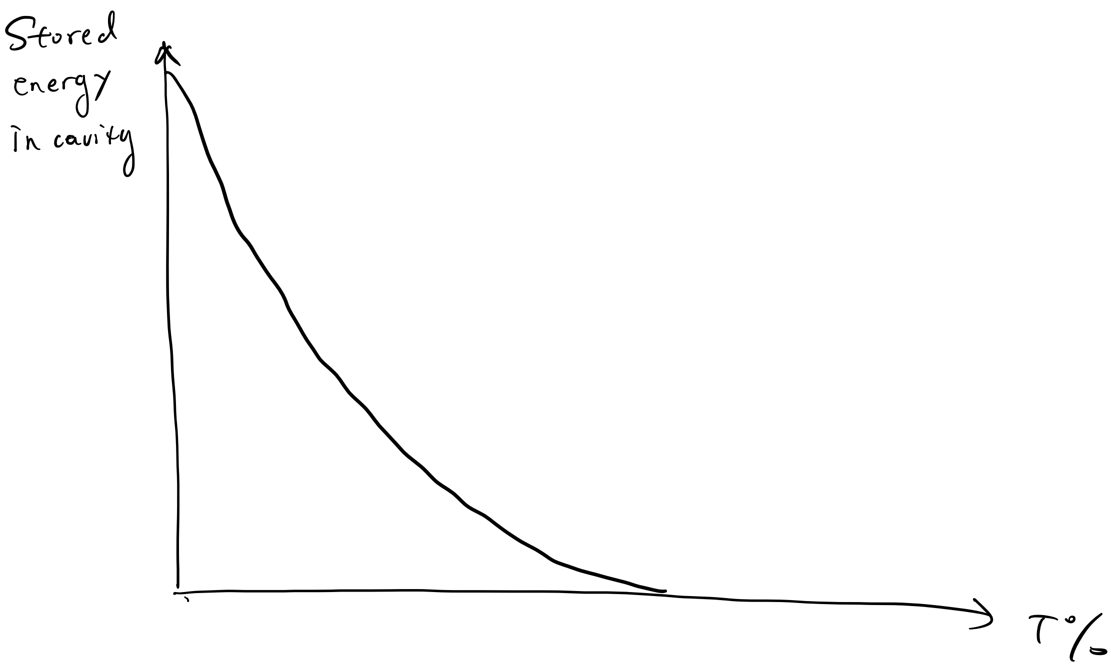
cross sectional area

$$P_o = I_s A T \left( \frac{g_o}{L_i + T} - 1 \right)$$

$$\frac{\partial P_o}{\partial T} = 0 \ni T_{opt} = -L_i + \sqrt{g_o L_i}$$

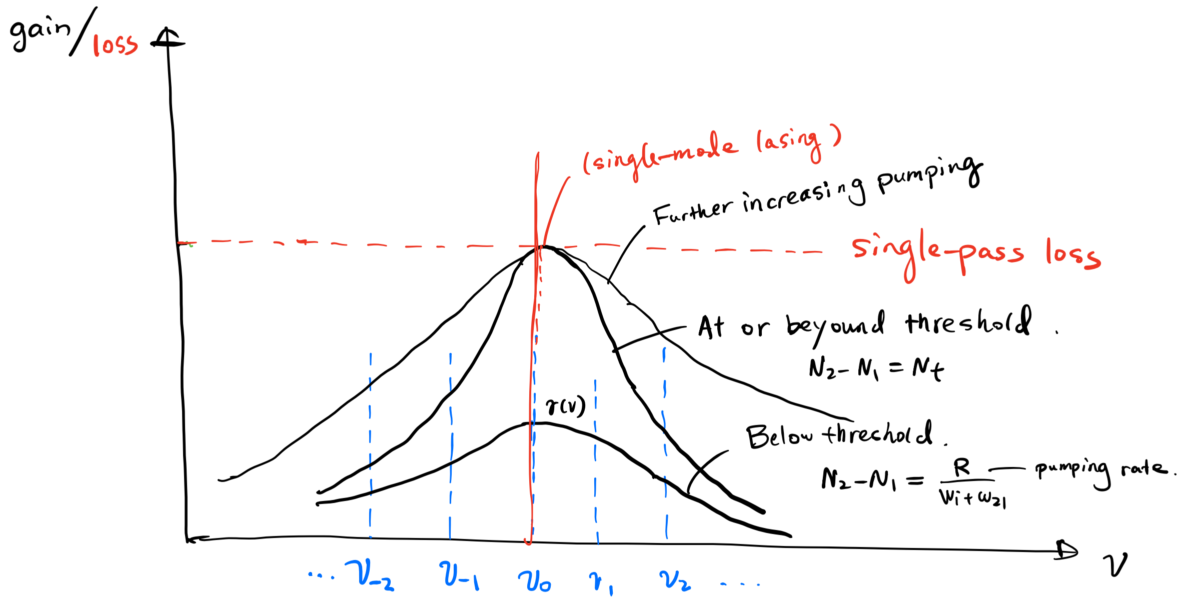
$$(P_0)_{opt} = I_s \cdot A \left(\sqrt{g_0} - \sqrt{L_i}\right)^2$$



## 2. Muti-mode lasing (p252 Yariv)

### 1) Homogeneous broadened gain medium.




$$\nu_{q+1} - \nu_q = \frac{c}{2nl} \quad (\text{FSR})$$

**Comments:**

For homogeneously broadened gain medium.

① Below threshold, population inversion $N_2 - N_1$ is proportional to pumping rate.

gain $\gamma(\nu) = (N_2 - N_1) \dfrac{c^2}{8\pi n^2 \nu^2 t_{spon}} g(\nu)$

② As pumping rate is increased, $N_2 - N_1$ is increased,

When $\gamma(\nu_0) = \gamma_t = \alpha - \frac{1}{l} \ln r_1 r_2$, lasing occurs.

③ Further increasing the pumping will **NOT** cause $\gamma(\nu_0)$ to increase. (gain saturation, or gain clamping). It will only lead to an increase of output intensity. i.e.

$$P_e = P_s \left( \frac{R}{R_t} - 1 \right) \quad \text{(last lecture)}$$

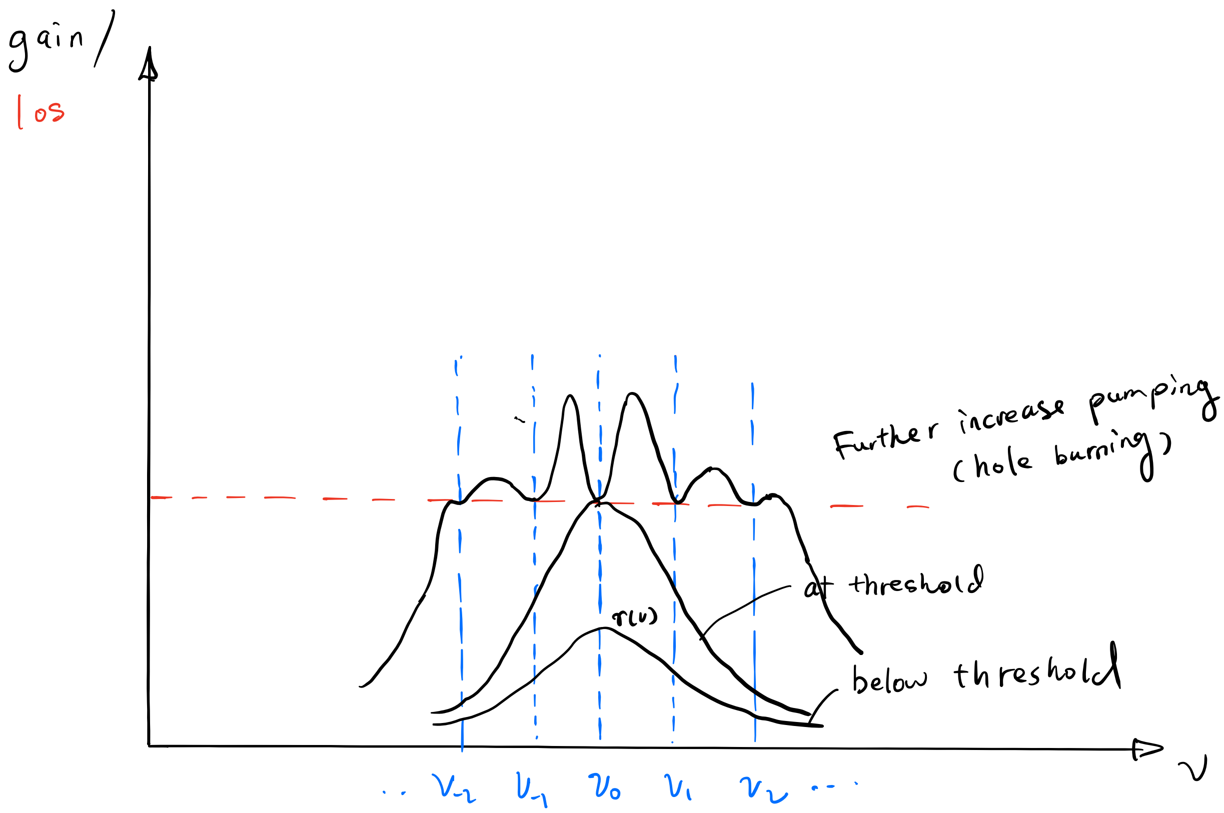
④ Further increasing the pumping will increase the intracavity power, eventually causing an additional broadening of $\gamma(\nu)$ due to shortening of lifetime by induced transmission.

The gain at other frequencies. ($\nu_{-1}$ $\nu_{-2}$, $\nu_{-1}$. $\nu_1$ ...) remain below threshold.

⑤ Ideal homogeneously broadened laser can only oscillate at a single mode.

## 2) Inhomogeneously broadened gain medium.

Extreme inhomogeneous case: the individual atoms can be regarded as being all different from each other.




## Comments:

① Below threshold, behavior is same as homogenously broadened case.

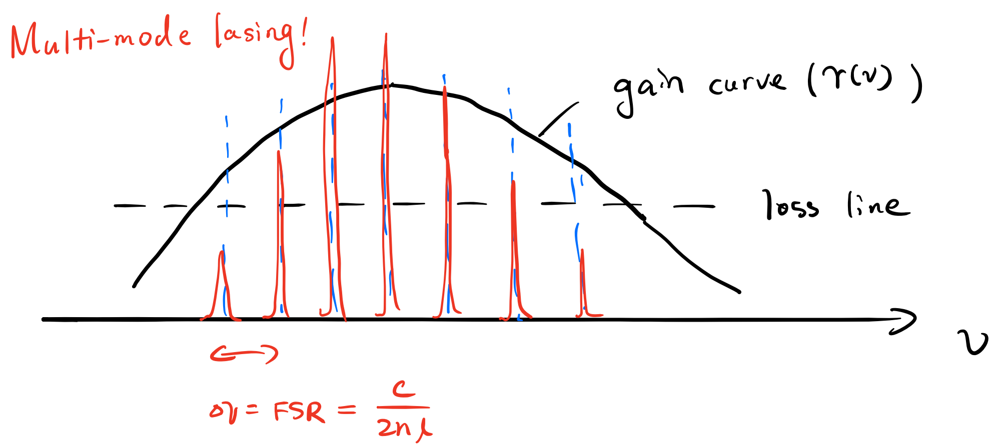
② At threshold, the gain at $\nu_0$ is clamped at the threshold value.

③ Due to inhomogeous broadening, atoms "do not communicate with each other". There's no reason why gain at other frequncies should not increase with further pumping.

④ Further pumping will lead to oscillation at additional longitudinal modes.

⑤ Gain at each oscillating frequency is clamped.

The gain profile curve aquires depressions at the oscillation frequencies. (a.k.a. "hole burning")




Phase of each lasing mode is radom!!

For inhomogenously broadened laser, oscillation can occur at different longitudinal modes:

$$\nu_q - \nu_{q-1} = FSR. = \frac{c}{2n\ell}$$

$$\Rightarrow \quad \omega_q - \omega_{q-1} = 2\pi\left(\nu_q - \nu_{q-1}\right) = \frac{\pi c}{n\ell}. \equiv \Omega$$

So the total output E-field. is

$$E(t) = \sum_m C_m e^{i[(\omega_0 + m\Omega)t + \phi_m]} \qquad 1$$

phase of the $m_{th}$ mode.

Amplitude of the $m_{th}$ mode

<u>Note</u>: If $\phi_m$ is fixed, $E(t)$ is periodic in time, period $= \frac{2\pi}{\Omega} = \frac{2\ell}{c}$

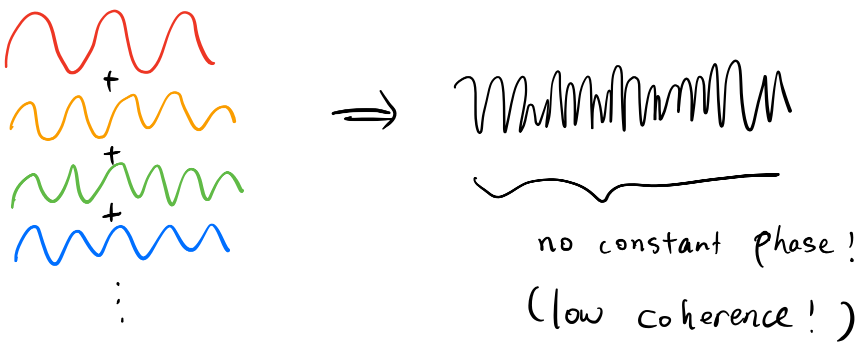
$$E(t+\tau) = \sum_m C_m \exp\left\{ i\left(\omega_0 + m\Omega\right)\left(t + \frac{2\pi}{\Omega}\right) + \phi_m \right\}$$

$$= \sum_m C_m \exp\left[i(\omega_0 + m\Omega)t + \phi_m\right] \exp i\left[2\pi\left(\frac{\omega_0}{\Omega} + m\right)\right]$$

$$= E_t \cdot \exp\left(\frac{i\, 2\pi \omega_0}{\Omega}\right)$$

But, in normal cases, $\phi_m$ is random!

Mode interefenence

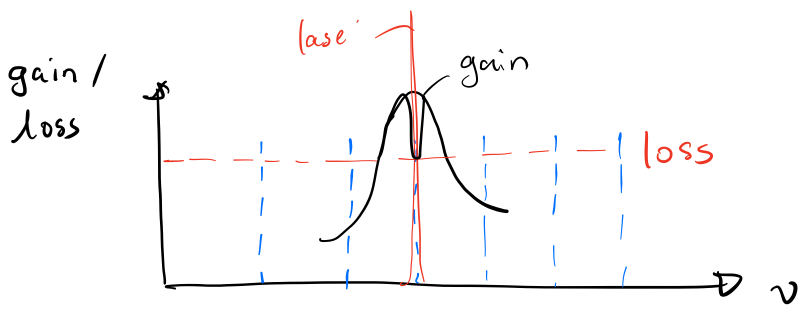



(show video!)

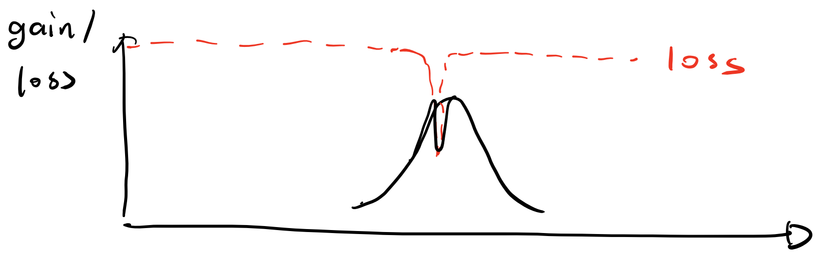
## Ways to get coherent output:

① Make cavity very small, mode spacing $\Omega = \frac{\pi c}{n\ell} \uparrow$.

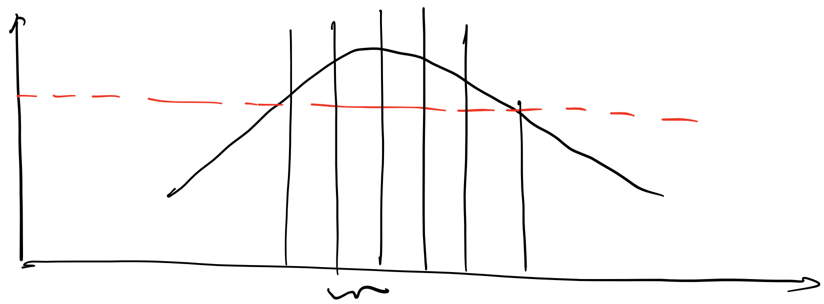
only one mode can lase!




② Make the reflector narrow band!



③ Mode-locking (P254. Yariv)




"lock" the phase between each mode.

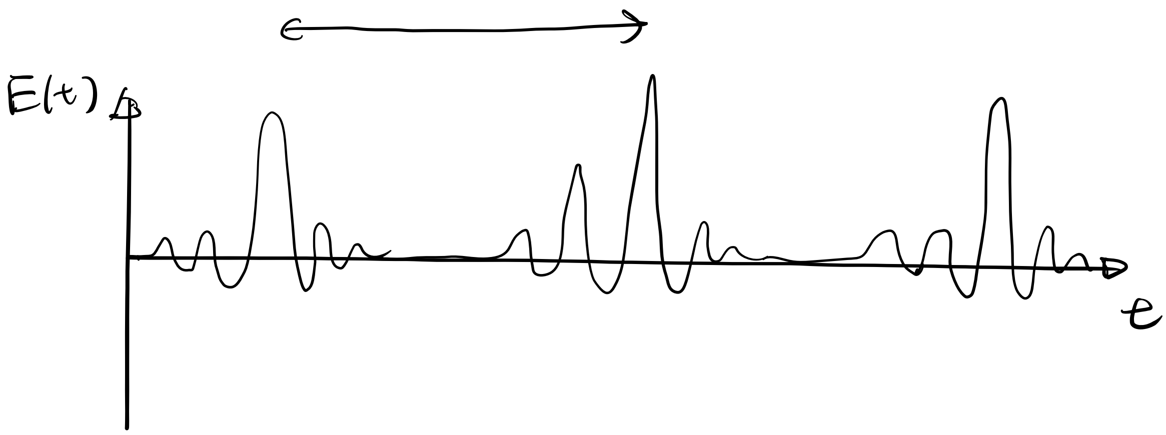
## 3. Mode-locking

Assuming $\underline{\phi_m = 0}$, $c_m = \frac{1}{\sqrt{N}}$, Eq. ① can be written as:

$$E(t) = \frac{1}{\sqrt{N}} \cdot \sum_{m=1}^{N} e^{i(\omega_0 + m\Omega)t}$$

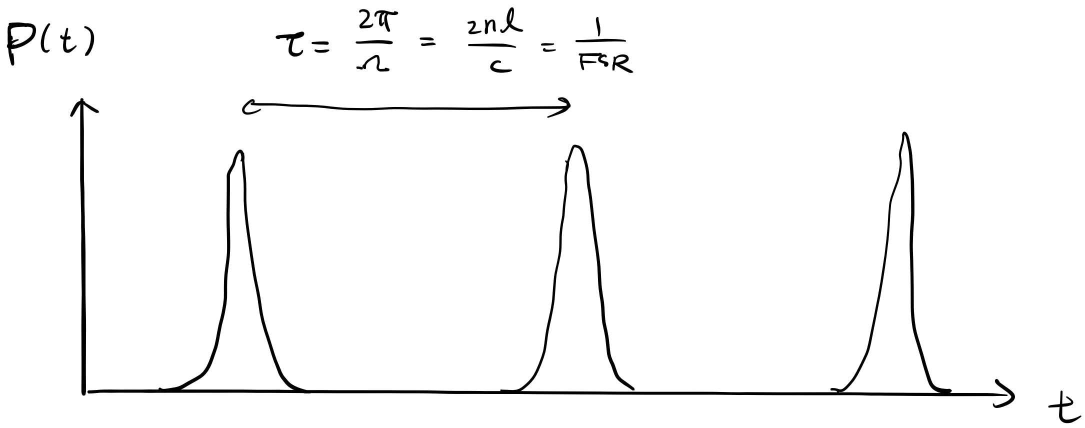
$$= \frac{1}{\sqrt{N}} e^{i[\omega_0 + (N+1)\Omega/2]t} \cdot \frac{\sin(N\Omega t/2)}{\sin(\Omega t/2)}$$

$$\tau = \frac{2\pi}{\Omega} = \frac{2nl}{c} = \frac{1}{FSR}$$



Power:

$$P(t) = E(t)\cdot E^*(t) \propto \frac{1}{N}\,\frac{\sin^2(N\Omega t/2)}{\sin^2(\Omega t/2)}$$




## Comments

If phase $\phi_m$ are "locked"

① The power is emitted in the form of a train of pulses, with a period of $\tau = \frac{2nl}{c}$. (the round trip transit time)

② The peak power of the pulse ($P(s\tau)$. $s = 0, 1, 2 \ldots$) is equal to N times the average power.

where N is the number of modes locked together.

③ The peak field amplitude is equal to N times the amplitude of a single mode

④ Pulse width (FWHM of $P(st)$) is $\tau_0 = \frac{\tau}{N}$.

$$N = \frac{\Delta\omega}{\Omega}$$

gain bandwith

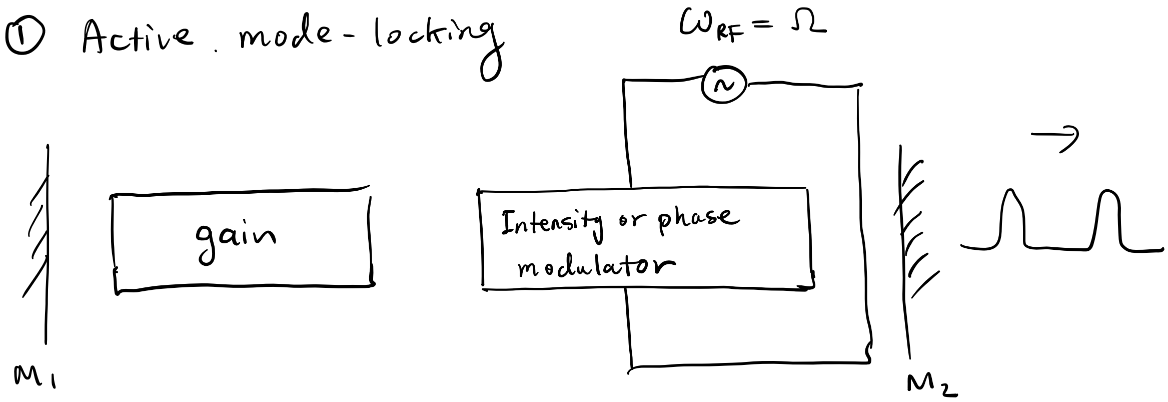
mode spacing.

$$\text{So} \quad \tau_0 = \frac{\tau}{N} = \frac{\frac{2nl}{c}}{\frac{\Delta\omega}{\Omega}} = \frac{2\pi}{\Delta\omega} = \frac{1}{\Delta\nu}$$

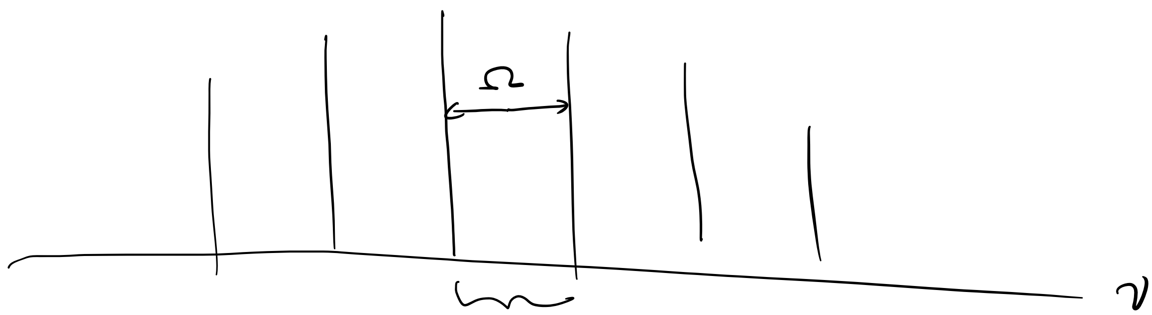
Physical meaning: to get shorter pulses, we need larger gain bandwidth!

## 4. Methods for mode-locking

① Active mode-locking

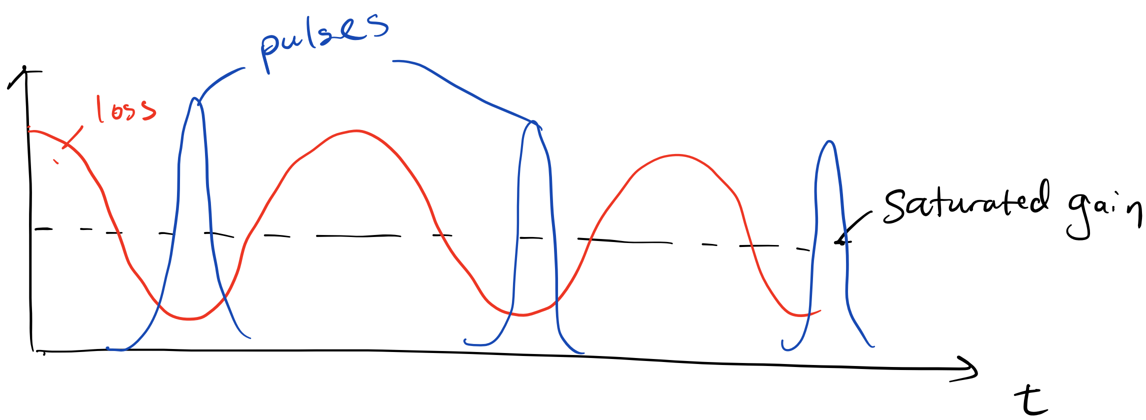



Freq. domain:

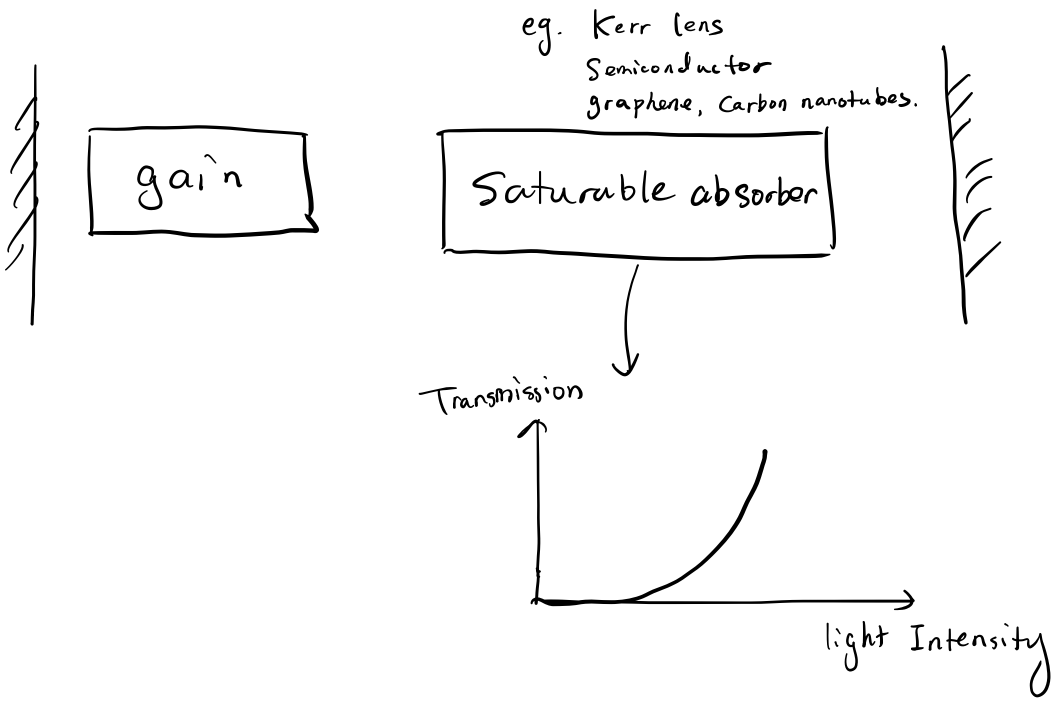



$\phi_m$ is "locked" by the RF drive

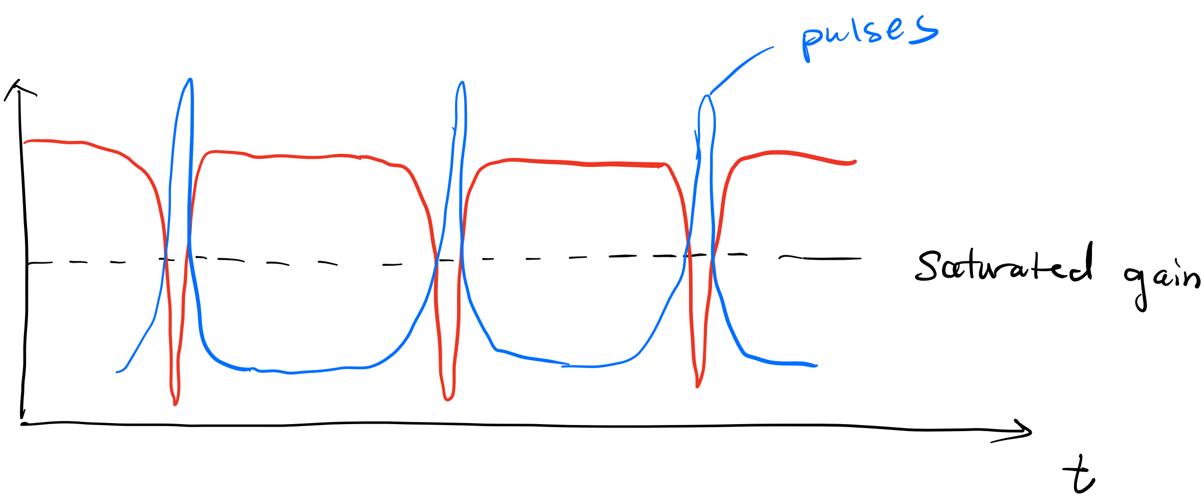
Time domain:



## ② Passive mode-locking




Time-domain




Note: Saturable absorber is a much faster "shutter" compared to external RF modulation. It allows for the generation of shorter pulses.