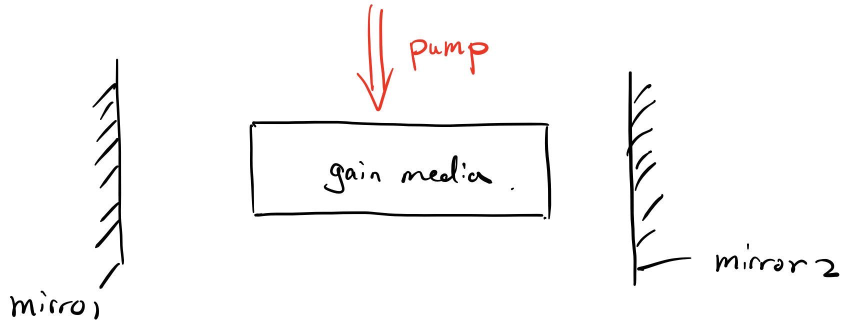
Lecture 15. Laser oscillation I.

Learning objectives:

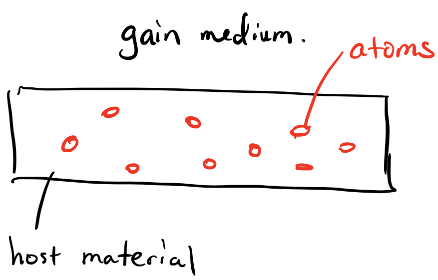
1. Lasers
2. Laser threshold
3. Oscillation frequency (frequency pulling)
4. Laser power

1. Lasers.



Three elements: ① gain medium, ② pump. ③ cavity.

2. Laser threshold (P 237 Yariv)



Complex refractive index:

$$n'^2 = \underbrace{n^2}_{\text{host}} + \underbrace{\chi' - i\chi''}_{\text{atoms.}}$$

Complex wavevector of plane waves in the gain medium:

$$k'(\omega) = k_0 n' = k_0 \sqrt{n^2 + \chi' - i\chi''}$$

For typical laser gain media, $|\chi' - i\chi''| \ll n^2$,

also, assuming α is the loss of the gain medium due to scattering at imperfections, absorption by excited atomic levels.

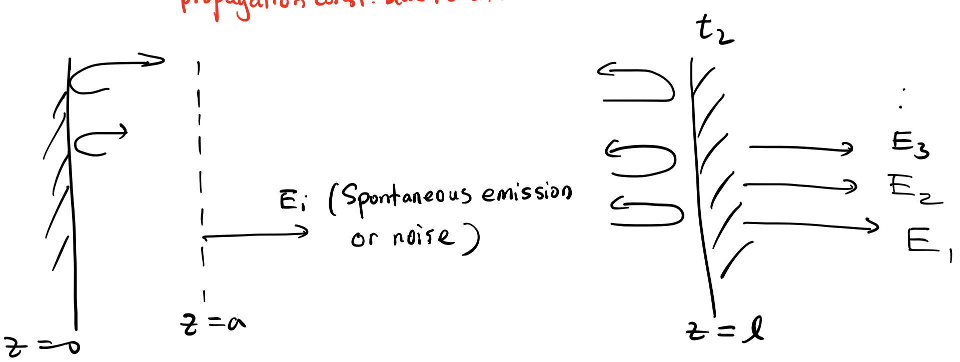
$$k'(\omega) = k_0 \sqrt{n^2 + \chi' - i\chi''} - i\frac{\alpha}{2} \simeq k + k\frac{\chi'(\omega)}{2n^2} - ik\frac{\chi''(\omega)}{2n^2} - i\frac{\alpha}{2}$$

$$= k + \Delta k + i(\gamma - \alpha)/2$$

where $k = n\frac{\omega}{c}$, $\Delta k = k\frac{\chi'(\omega)}{2n^2}$, $\gamma = -k\frac{\chi''(\omega)}{n^2} = (N_2 - N_1)\frac{\lambda^2}{8\pi n^2 t_{spon}} g(\nu)$

Correction to the propagation const. due to atoms.

F.P. cavity



$$E_1 = t_2 e^{-ik'(\ell-a)} E_i$$

$$E_2 = (r_1 r_2 e^{-2k'\ell}) t_2 e^{-ik'(\ell-a)} E_i$$

$$E_3 = (r_1 r_2 e^{-2k'\ell})^2 t_2 e^{-ik'(\ell-a)} E_i$$

$$E_4 = (r_1 r_2 e^{-2k'\ell})^3 t_2 e^{-ik'(l-a)} E_i$$

$$E_{out} = E_1 + E_2 + E_3 + E_4 + \cdots$$

$$= \frac{t_2 e^{-ik'(l-a)}}{1 - r_1 r_2 e^{-2k'l}} E_i$$

plug in k', we get:

$$E_{out} = \frac{t_2 e^{-i[(k+\Delta k)(l-a)]} e^{(\gamma-\alpha)(l-a)/2}}{1 - r_1 r_2 e^{-i2(k+\Delta k)l} e^{(\gamma-\alpha)l}} E_i$$

Comments:

1. If population is inverted, (i.e. $N_2 > N_1$, $\gamma > 0$) the denominator can become very small. E_{out} can be larger than E_i

2. When $r_1 r_2 e^{-i2(k+\Delta k)l} e^{(\gamma-\alpha)l} = 1$, $E_{out}/E_i \to \infty$.

   This corresponds to a finite E_{out} with $E_i \simeq 0$.

   (oscillation or lasing!)

Laser oscillating condition:

$$\boxed{r_1 r_2 e^{-i2(k+\Delta k)\ell} e^{(\gamma-\alpha)\ell} = 1.} \qquad (1)$$

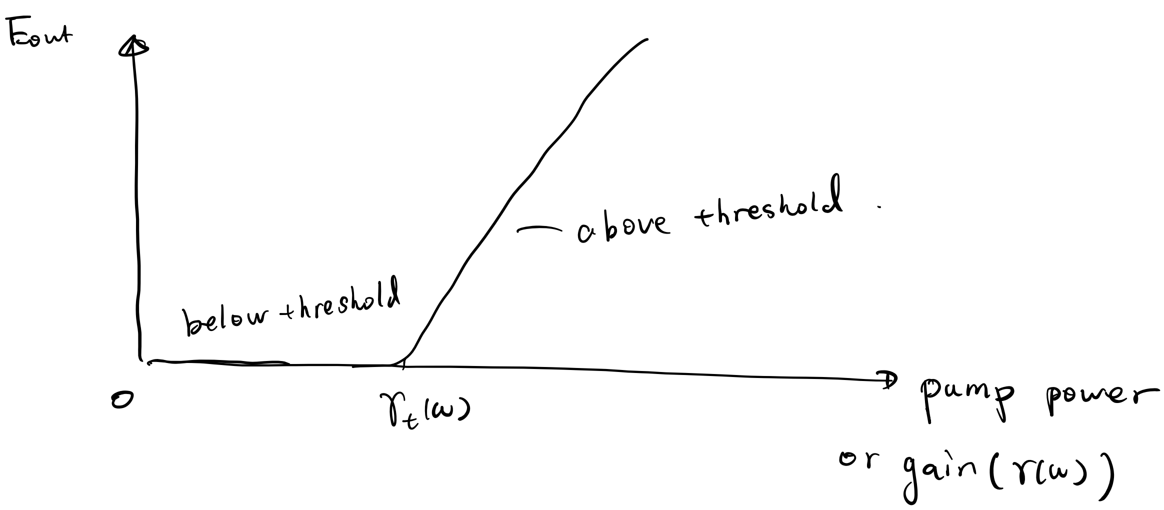
To satisfy. we have:

Amplitude: $r_1 r_2 e^{[\gamma(\omega)-\alpha]\ell} = 1$ (2)

phase: $2[k+\Delta k(\omega)]\ell = 2m\pi$, $m = 1, 2 \ldots$ (3)

To satisfy (2), threshold gain coefficient.

$$\boxed{\gamma_t(\omega) = \alpha - \frac{1}{\ell} \ln r_1 r_2}$$



Population inversion density at threshold:

$$N_t = (N_2 - N_1)_t = \frac{8\pi n^2 t_{spon}}{g(\nu)\lambda^2}\left(\alpha - \frac{1}{\ell}\ln r_1 r_2\right)$$

3. Oscillation frequency and frequency pulling

Let's look at the phase part.

$$2(k+\Delta k)\ell = 2m\pi \qquad m = 1, 2, \dots$$

Meaning: many modes can satisfy this phase condition. If the gain condition (eq. ③) can be satisfied, these modes can lase.

$$k\ell\left(1+\frac{\chi'(\nu)}{2n^2}\right) = m\pi \qquad ④$$

For a "cold cavity" (without gain media), the Mth resonance frequency is $\nu_m = \dfrac{mc}{2n\ell}$ ← length of cavity

Also, $\chi'(\nu)$ and $\chi''(\nu)$ are related by

$$\chi'(\nu) = \frac{2(\nu_0-\nu)}{\Delta\nu}\chi''(\nu), \quad = \frac{2(\nu_0-\nu)}{\Delta\nu}\cdot\left(-\frac{n^2\gamma(\nu)}{k}\right)$$

Eq. ④ can be rewritten as:

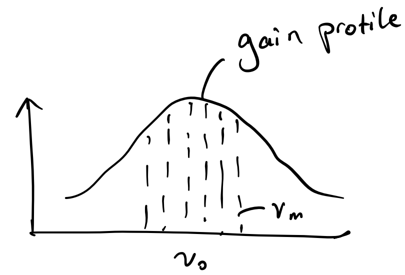
$$\nu\left[1-\left(\frac{\nu_0-\nu}{\Delta\nu}\right)\frac{\gamma(\nu)}{k}\right] = \nu_m \qquad ⑤$$

where ν_0 is the center of the atomic lineshape function (gain profile)

Assuming we can tune the cavity such that

$\nu_m \rightarrow \nu_0$.

oscillation frequency $\nu \rightarrow \nu_0$.



Note: ν_m is "cold cavity" freqs.
ν is lasing freqs.
$\nu \neq \nu_m$.

Eq. 5 can be rewritten as

$$\nu = \nu_m - (\nu_m - \nu_0)\frac{\gamma(\nu_m)\,c}{2\pi n\,\Delta\nu} \qquad (6)$$

let's simplify further!

Threshold gain:

$$\gamma_t(\nu) = \alpha - \frac{1}{\ell} r_1 r_2$$

Let $r_1 = r_2 = \sqrt{R}$. assuming $R \sim 1$. $\alpha = 0$.

$$\gamma_t(\nu) \simeq \frac{1-R}{\ell}$$

Recall the "cold cavity" resonance linewidth:

$$\Delta\nu_{1/2} \simeq \frac{c(1-R)}{2\pi n \ell}$$

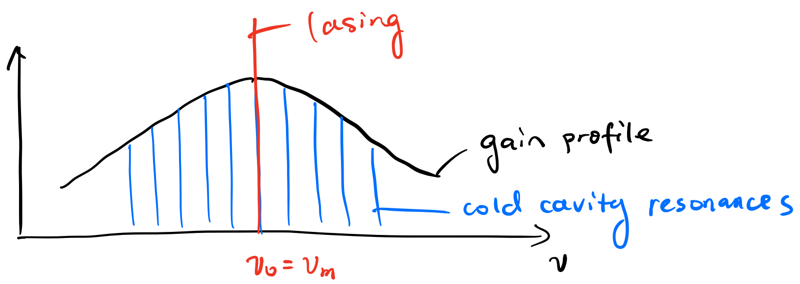
Eq. (6) can be re-written as:

$$\boxed{\nu = \nu_m - (\nu_m - \nu_0)\frac{\Delta\nu_{1/2}}{\delta\nu}}$$

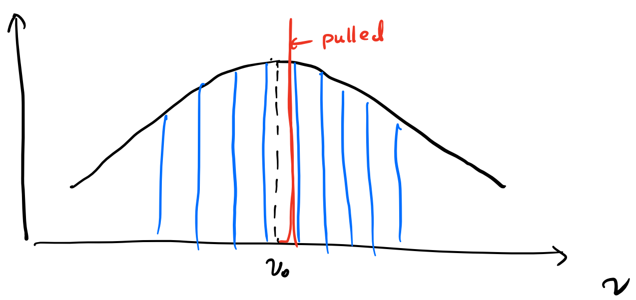
laser oscillation freq.

Comments:

① When "cold cavity" resonance coincides with the atomic line center, ($\nu_m = \nu_0$), oscillation occurs at $\nu = \nu_0$

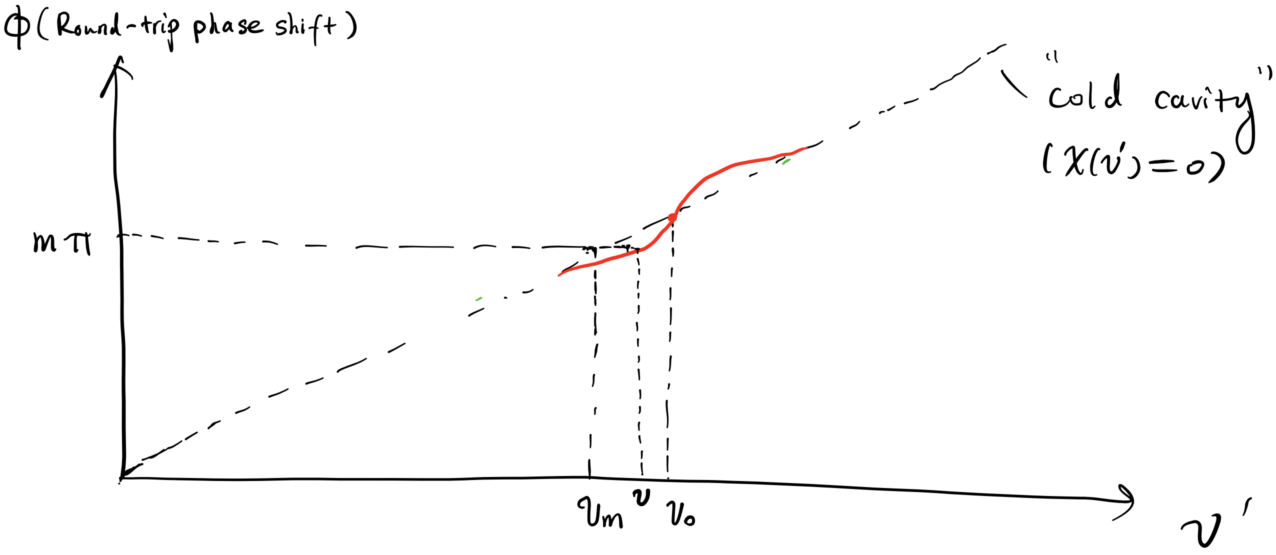


② When $\nu_m \neq \nu_0$. oscillation occurs at ν_m but is "pulled" slightly to ν_0



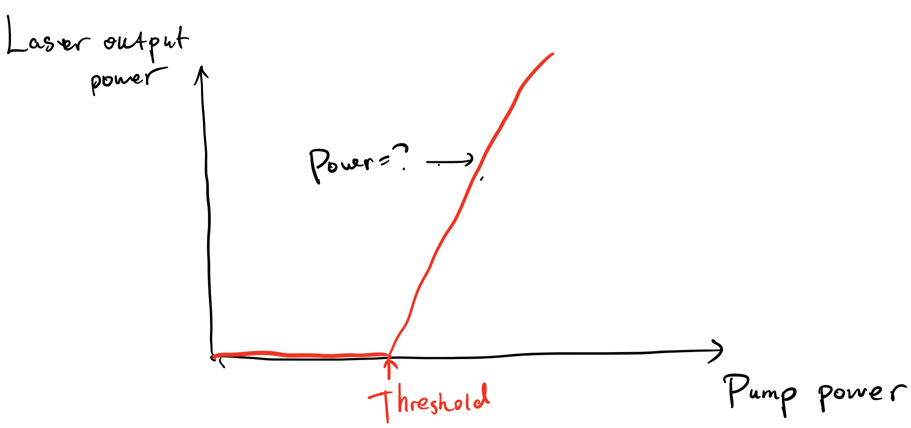
③ Reason for "frequency pulling":

Atomic dispersion $\chi'(\nu)$ "pulls" the laser oscillation freq. ν. from the passive resonator value ν_m toward atomic resonance at ν_0

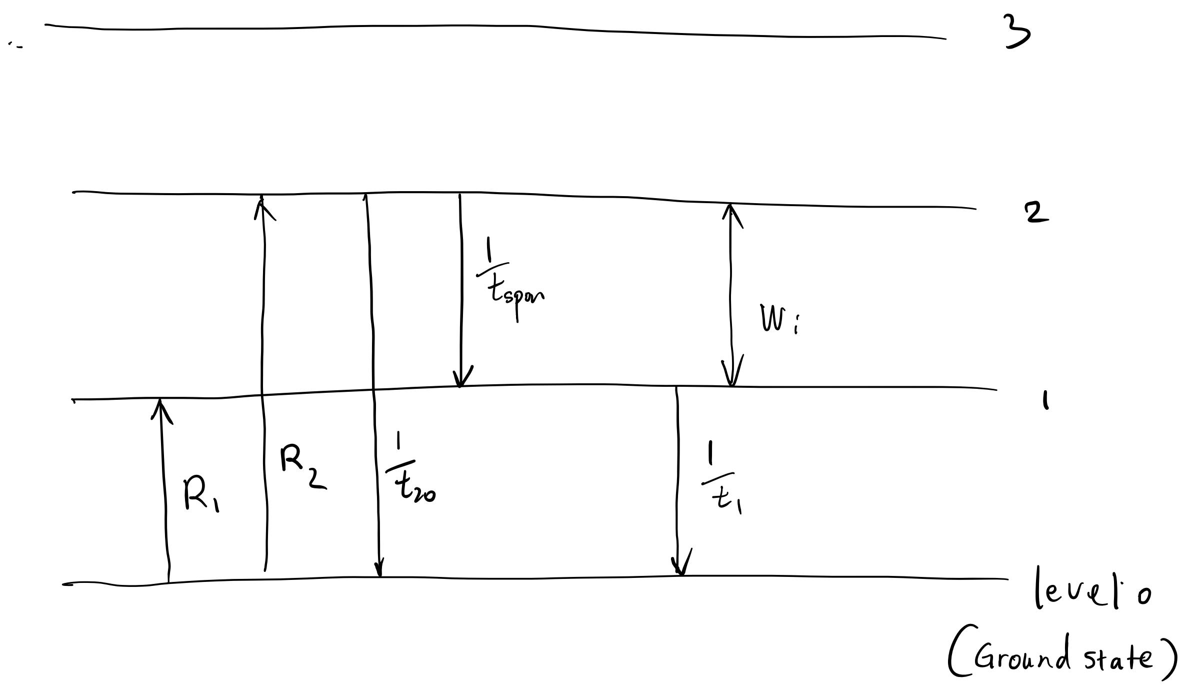


4. Powers in laser oscillators

Probem to solve here:



Recall Four-level system.



Rate equations:

$$\begin{cases} \dfrac{dN_2}{dt} = R_2 - \dfrac{N_2}{t_2} - (N_2 - N_1)\underline{W_i(\nu)} \\ \dfrac{dN_1}{dt} = R_1 - \dfrac{N_1}{t_1} + \dfrac{N_2}{t_{spon}} + \underbrace{(N_2 - N_1) W_i(\nu)} \end{cases}$$

Stimulated emission

Stimulated emission

where $\frac{1}{t_2} = \frac{1}{t_{spon}} + \frac{1}{t_{20}}$

Assuming spontaneous emission from $2 \to 1$ is dominant,

$t_2 \sim t_{spon}$, replace $\frac{1}{t_2}$ by w_{21} , replace $\frac{1}{t_{spon}}$ by ω_{21}

replace $\frac{1}{t_1}$ by ω_{10}

$$\begin{cases} \dfrac{dN_2}{dt} = -N_2 \omega_{21} - W_i (N_2 - N_1) + R_1 \\[2ex] \dfrac{dN_1}{dt} = -N_1 \omega_{10} + N_2 \omega_{21} + W_i (N_2 - N_1) + R_1 \end{cases}$$

In steady state. $\dfrac{dN_2}{dt} = 0$, $\dfrac{dN_1}{dt} = 0$,

Effective pumping rate.

$$\boxed{N_2 - N_1 = \frac{R_2 \left[1 - (\omega_{21}/\omega_{10})(1 + R_1/R_2) \right]}{W_i + \omega_{21}} = \frac{R}{W_i + W_{21}}}$$

To achive population inversion, $\omega_{21} < \omega_{10}$

<u>Meaning</u>: upper state liftime > lower state lifetime.

Three regimes:

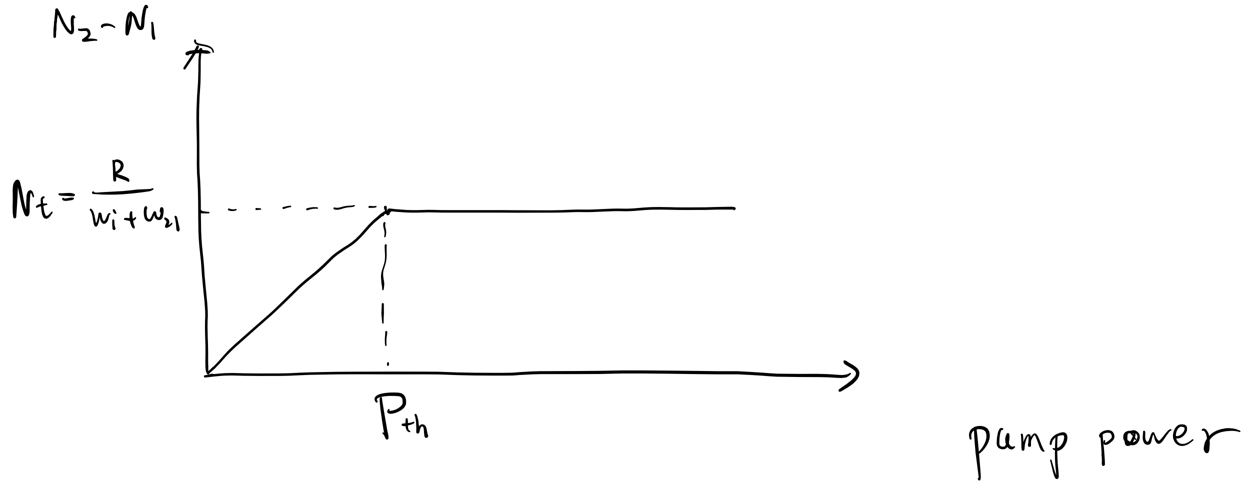
① Below threhold, induced transition rate $W_i \simeq 0$

$$N_2 - N_1 = \frac{R_2}{W_{21}}$$

② At threshod, $N_2 - N_1 = N_t = \frac{8\pi n^3 \nu^2 t_{spon}}{g(\nu)\lambda^2}\left(\alpha - \frac{1}{l}\ln r_1 r_2\right)$

gain = cavity round-trip loss

③ Beyond threshold, $N_2 - N_1$ cannot further increase due to gain saturation. If it still increases, W_i > loss, violates the steady-state assumption.



So $$N_t = \frac{R}{W_i + w_{21}}$$

$$\Rightarrow W_i = \frac{R}{N_t} - w_{21}$$

The total power generated by stimulated emission:

$$P_e = (N_t \overset{\text{volume}}{V}) W_i \, h\nu$$

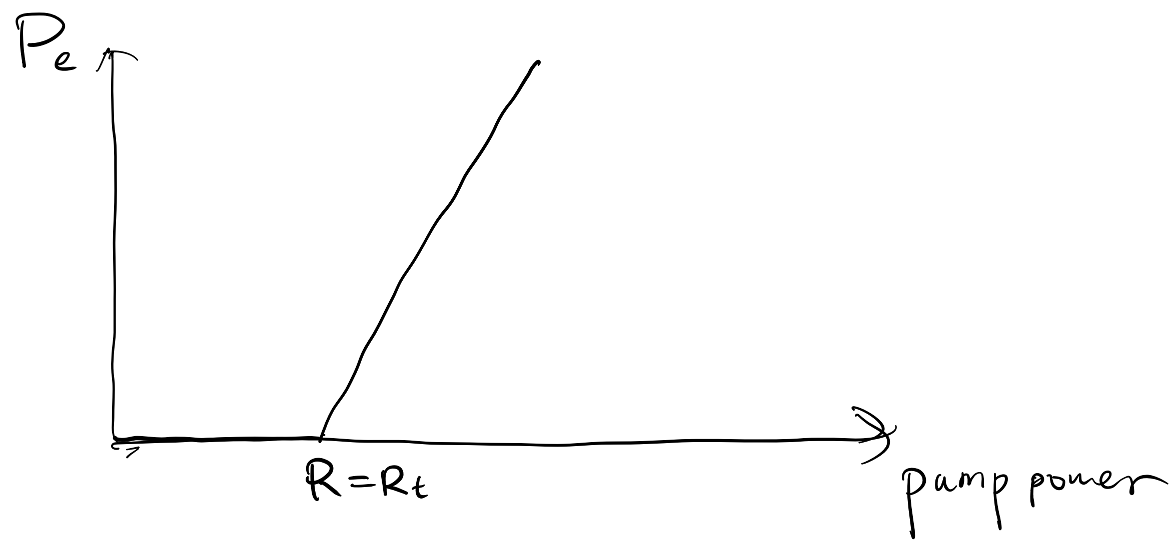
$$\Rightarrow P_e = v \cdot h\nu \cdot N_t \cdot \left(\frac{R}{N_t} - w_{21} \right)$$

$$= \boxed{v \, h\nu \, N_t \, w_{21}} \left(\frac{R}{N_t \cdot w_{21}} - 1 \right)$$

$$= P_s \left(\frac{R}{R_t} - 1 \right)$$

threshold pumping rate.

power going into spontaneous emission at thresold.



P_e
$R=R_t$
pump power