Lecture 14. Stimulated Brillouin Scattering

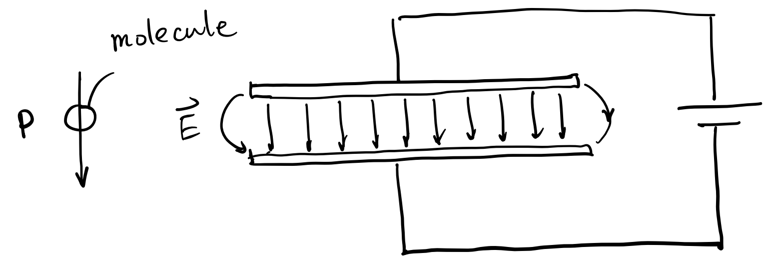
Learning objectives:

1. Electrostriction
2. SBS
3. Brillouin gain
4. SBS in integraled nanophotonics

1. Electrostriction

Definition: elastic compression in the presence of an EM field. (a 3rd order nonlinearity)

Intuitive picture: a molecule near the parallel plate capacitor



dipole moment on the molecule:

$$\vec{p} = \epsilon_0 \alpha \vec{E}$$

Energy stored in the polarization:

$$U = -\int_0^{\vec{E}} \vec{p} \cdot d\vec{E}' = -\int_0^{\vec{E}} \epsilon_0 \alpha \vec{E}' d\vec{E}' = -\frac{1}{2} \epsilon_0 \alpha E^2$$

Force acting on the molecule:

$$\vec{F} = -\nabla U = \frac{1}{2} \epsilon_0 \alpha \nabla (|E|^2)$$ → gradient forces

How about a dielectric medium instead of a dipole?

Applied field $\Rightarrow$ molecular arrangement $\Rightarrow$ change in density

$\Rightarrow$ change in susceptibility (or ϵ, n)

$$\epsilon = 1 + \alpha N$$

Raman   Brillouin

$$\Delta\epsilon = \frac{\partial\epsilon}{\partial\rho}\cdot\Delta\rho$$

$$= \underbrace{\left(\rho\frac{\partial\epsilon}{\partial\rho}\right)}_{\gamma_e = \text{electrostrictive const.}}\cdot\frac{\Delta\rho}{\rho} = \gamma_e\frac{\Delta\rho}{\rho}$$

will show that $\gamma_e = \frac{1}{3}(n^2-1)(n^2+2)$

$$\Delta\rho = \frac{1}{2}\epsilon_0\,\rho\,C_T\,\gamma_e\cdot\langle\tilde{\vec{E}}(t)\cdot\tilde{\vec{E}}(t)\rangle$$

$$\text{Compressibility} = -\frac{1}{V}\left.\frac{\partial V}{\partial p}\right|_{\text{const } T} = \frac{1}{\rho}\cdot\frac{\partial\rho}{\partial p} > 0$$

For monochromatic input field:

$$\vec{E}(t) = A e^{-i\omega t} + c.c.$$

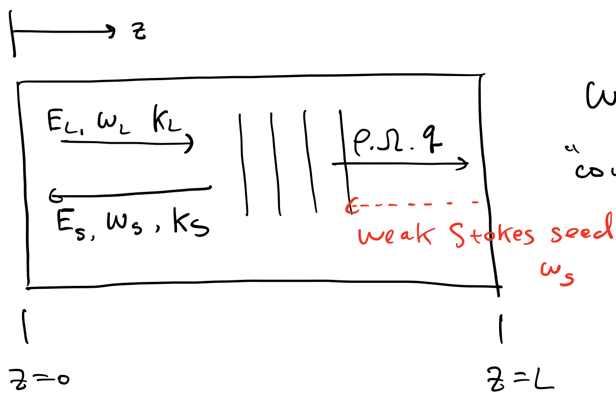
$$\langle\tilde{\vec{E}}(t)\cdot\tilde{\vec{E}}(t)\rangle = 2A\cdot A^* = 2|A|^2$$

So $\Delta\epsilon = \underbrace{\epsilon_0 C_T \gamma_e^2}_{\chi^{(3)}_{\text{Brillouin}}} |A|^2$

Input multiple freq & get beating in E·E can excite acoustic wave at beat freq!

2. Stimulated Brillouin Scattering (SBS)

Physical picture:



ω_L and ω_s are "counter-propagating"!

① When incident light is of high intensity, even spontaneously scattered light can be quite intense.

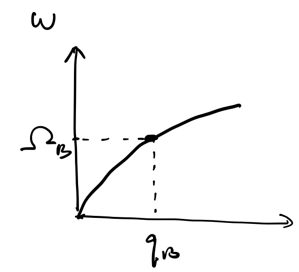
② The incident and scattered light fields beat together, giving rise to acoustic wave and index variation due to electrostriction ⇒ More scattered light at ω_s

③ Acoustic wave and ω_s will mutually reinforce each other's growth.

Math:

$$\omega_S = \omega_L - \Omega_B \quad (1)$$
will be determined.

$\Omega_B = |\vec{q}_B| \cdot v \rightarrow$ velocity of sound.



Dispersion of acoustic wave

where $\vec{q}_B = \vec{k}_L - \vec{k}_S = n\frac{\omega_L}{c} - \left(-n\frac{\omega_S}{c}\right)$

$$= \frac{n}{c}(\omega_L + \omega_S)$$

$$\Rightarrow \Omega_B = \frac{v}{c/n}(\omega_L + \omega_S) \quad (2)$$

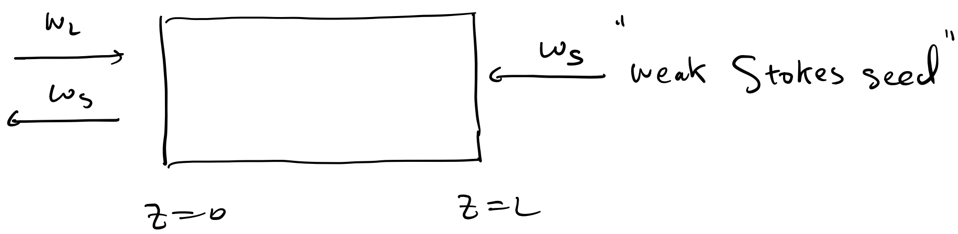
Solve (1). (2) together.

$$\Omega_B = \frac{\frac{2v}{c/n}\omega_L}{1 + \frac{v}{c/n}} \overset{v \ll c/n}{\simeq} \frac{2v}{c/n}\omega_L$$

Also, $k_1 \simeq k_2$. So

$$\vec{q}_B \simeq 2\vec{k}_L$$

Now, consider SBS amplifer with pump (ω_L) and Stokes seed "ω_s" as inputs



The driven acoustic wave due to beat note:

$$\Omega = \omega_L - \omega_s$$ → will compare Ω and Ω_B later!

Three waves:

$$\tilde{E}_L(z,t) = A_L(z,t)\, e^{i(k_L z - \omega_L t)} + c.c.$$

$$\tilde{E}_S(z,t) = A_S(z,t)\, e^{i(-k_S z - \omega_S t)} + c.c.$$

$$\tilde{\rho}(z,t) = \rho_0 + [\rho(z,t)\, e^{i(qz - \Omega t)} + c.c.]$$ → material density wave.

where $\Omega = \omega_L - \omega_s$, $q = 2k_L$

Accoustic wave equation of material:

$$\frac{\partial^2 \tilde{\rho}}{\partial t^2} - \Gamma' \nabla^2 \frac{\partial \tilde{\rho}}{\partial t} - v^2 \nabla^2 \tilde{\rho} = \nabla \cdot \vec{f} \quad (1)$$

Γ' → damping

v → Sound velocity

$\vec{f} = \nabla p_{st}$. → Electristice pressure

$p_{st} = -\frac{1}{2}\epsilon_0 \gamma_e \langle \tilde{E}^2 \rangle$

$\tilde{E}$ → optical field

$$\nabla \cdot \vec{f} = \epsilon_0 \gamma_e q^2 [A_1 A_2^* e^{i(qz - \Omega t)} + c.c.] \quad (2)$$

Plug ② into ①, assume acoustic wave varies slowly in time & space, i.e. $\frac{\partial^2 \rho}{\partial t^2} \ll \frac{\partial \rho}{\partial t}$, $\frac{\partial^2 \rho}{\partial z^2} \ll \frac{\partial \rho}{\partial z}$, we get

$$-2i\Omega \frac{\partial \rho}{\partial t} + (\Omega_B^2 - \Omega^2 - i\Omega\Gamma_B)\rho - 2iqv^2 \frac{\partial \rho}{\partial z} = \epsilon_0 \gamma_e q^2 A_1 A_2^*$$

Brillouin linewidth

$\Gamma_B = q^2 \Gamma'$, phonon lifetime $\tau_p = \frac{1}{\Gamma_B}$

Assumption: ① phonon progates much shorter than light wave. $\frac{\partial \rho}{\partial z} = 0$

② Steady state. $\frac{\partial \rho}{\partial t} = 0$

$$\rho(z,t) = \epsilon_0 \gamma_e q^2 \cdot \frac{A_1 A_2^*}{\Omega_B^2 - \Omega^2 - i\Omega\Gamma_B}$$

$\Rightarrow$ Nonlinear polarization:

$$\tilde{P} = \epsilon_0 \Delta\chi \tilde{E} = \epsilon_0 \Delta\epsilon \tilde{E} = \epsilon_0 \gamma_e \frac{\rho - \rho_0}{\rho_0} E$$

$$= \frac{\epsilon_0 \gamma_e}{\rho_0} \left[\rho e^{i(qz - \Omega t)} + c.c.\right]\left[A_L e^{i(k_L z - \omega_L t)} + A_S e^{-(k_S z - \omega_S t)} + c.c.\right]$$

contains many frequencies!

Next step: pick up the polarization at ω_L and ω_S!

$$\begin{cases} \tilde{P}_L = \epsilon_0 \gamma_e \rho_0^{-1} \rho A_2 \\ \tilde{P}_S = \epsilon_0 \gamma_e \rho_0^{-1} \rho^* A_1 \end{cases} \Rightarrow \text{plug in NL wave equation:}$$

$$\frac{\partial^2 E_i}{\partial z^2} - \frac{1}{(c/n)^2} \frac{\partial^2 E_i}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2 \tilde{P}_i}{\partial t^2}$$

$\omega_L \simeq \omega_S = \omega$

$$\Rightarrow \begin{cases} \dfrac{\partial A_L}{\partial z} + \dfrac{1}{c/n} \dfrac{\partial A_L}{\partial t} = \dfrac{i\omega \gamma_e}{2nc\rho_0} \rho A_S \\ -\dfrac{\partial A_S}{\partial z} + \dfrac{1}{c/n} \dfrac{\partial A_S}{\partial t} = \dfrac{i\omega \gamma_e}{2nc\rho_0} \rho^* A_L \end{cases}$$

Steady-state CAE:

$$\frac{dA_L}{dz} = \frac{i\epsilon_0 \omega q^2 \gamma_e^2}{2nc\rho_0} \frac{|A_S|^2 A_L}{\Omega_B^2 - \Omega^2 - i\Omega\Gamma_B}$$

$$\frac{dA_S}{dz} = \frac{-i\epsilon_0 \omega q^2 \gamma_e^2}{2nc\rho_0} \frac{|A_L|^2 A_S}{\Omega_B^2 - \Omega^2 + i\Omega\Gamma_B}$$

3. Brillouin gain & amplification

Intensity: $I = 2n\epsilon_0 c A \cdot A^*$

CAE can be written as:

$$\begin{cases} \dfrac{dI_L}{dz} = -g I_1 I_2 & (1) \\[2ex] \dfrac{dI_S}{dz} = -g I_1 I_2 & (2) \end{cases}$$

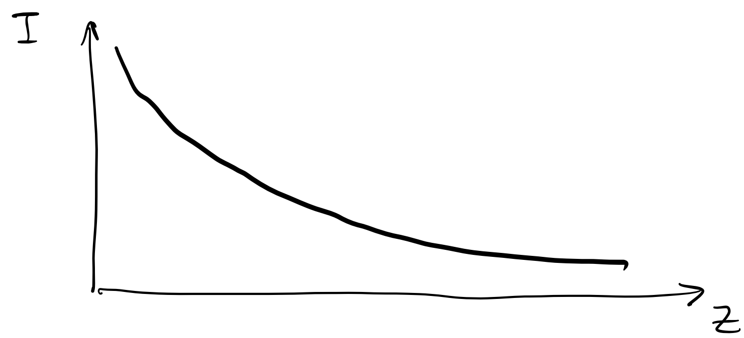
SBS gain factor: $g = g_0 \dfrac{(\Gamma_B/2)^2}{(\Omega_B - \Omega)^2 + (\Gamma_B/2)^2}$

$$g_0 = \frac{\gamma_e^2 \omega^2}{n v c^3 \rho_0 \Gamma_B}$$

Assume no pump depletion: $I_L = C$, solution of (2) is

$$I_S(z) = I_S(L)\, e^{g I_1 (L - z)}$$

Stokes wave injected at $z = L$ experiences exponential growth! (negative sign of g means exp. growth backward)



Peak Brillouin gain:

$$g_B(\Omega = \Omega_B) \propto \frac{\gamma_e^2 q^2}{\rho_0^2} \frac{1}{\Omega \Gamma_B}$$

Compare Brillouin to Raman:

$$\frac{g_B(\Omega_B)}{g_R(\Omega_R)} = (\text{other factors}) \times \underbrace{\frac{\Gamma_B^{-1}}{\Gamma_R^{-1}}}$$

$$\frac{\tau_B}{\tau_R} \simeq \frac{10\,\text{ns}}{50\,\text{fs}} = 2 \times 10^5$$