Lecture 11. Surface plasmon polaritons

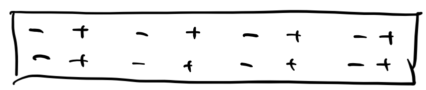
1. Surface plasmon polaritons (spp) at a single interface

2. SPP in multi-layer system

Recap:

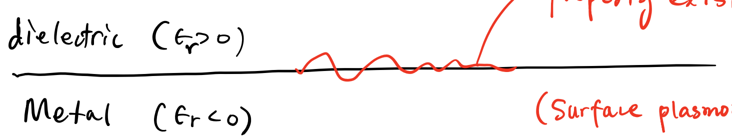
For metals $\epsilon_r(\omega) = 1 - \dfrac{\omega_p^2}{\omega^2 + i\gamma\omega}$

① when $\omega = \omega_p$. bulk plasmons (longitudinal wave)



② when $\omega < \omega_p$, $\epsilon_r(\omega) < 0$.

A surface wave with unique property exist!



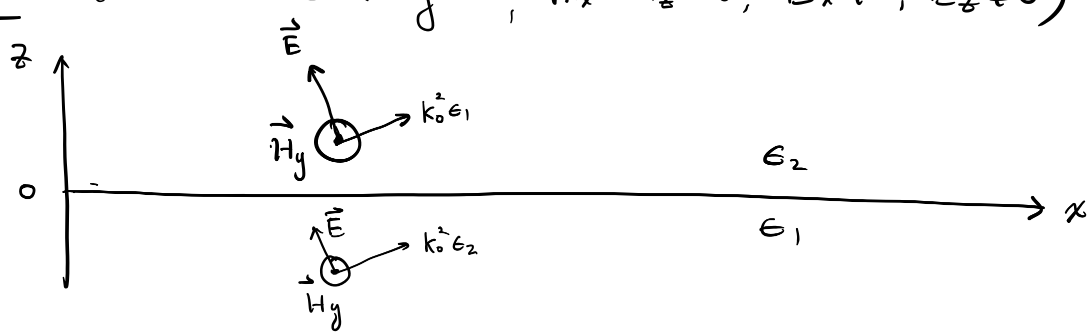
(Surface plasmon polariton (SPP))

1. Surface plasmon polaritons at a single interface

problem to solve here:

① find if there is a wave propagating along x.
② find the dispersion relation of the wave

Case I: TM wave ($H_y \neq 0$, $H_x = H_z = 0$, $E_x \neq 0$, $E_z \neq 0$)



For $z > 0$

$$
\begin{cases}
H_y(z) = A_2 \underbrace{e^{i\beta x}}_{\text{propagating}} \underbrace{e^{-k_2 z}}_{\text{decay}} \\
E_x(z) = -i \dfrac{1}{\omega \epsilon_0 \epsilon} \dfrac{\partial H_y}{\partial z} = i A_2 \dfrac{1}{\omega \epsilon_0 \epsilon_2} k_2 e^{i\beta x} e^{-k_2 z} \\
E_z(z) = -\dfrac{\beta}{\omega \epsilon_0 \epsilon} H_y = -A_2 \dfrac{\beta}{\omega \epsilon_0 \epsilon_2} e^{i\beta x} e^{-k_2 z}
\end{cases}
$$

For $z < 0$

$$\begin{cases} H_y(z) = A_1 e^{i\beta x} e^{k_1 z} \\ E_x(z) = -i A_1 \dfrac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta x} e^{k_1 z} \\ E_z(z) = -A_1 \dfrac{\beta}{\omega \epsilon_0 \epsilon_1} e^{i\beta x} e^{k_1 z} \end{cases}$$

B.C. H_y and E_x are continuous

$$\Rightarrow \begin{cases} A_1 = A_2 \\ \boxed{\dfrac{k_2}{k_1} = -\dfrac{\epsilon_2}{\epsilon_1}} \end{cases} \qquad ①$$

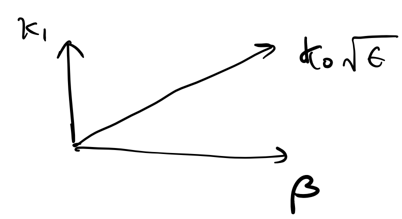
Note: when ϵ_2 and ϵ_1 are of opposite signs, k_2 and k_1 can be both real & positive!

Meaning: A surface wave exist between metal and dielectric.

Also, the wave eq. for TM wave is

$$\frac{\partial^2 H_y}{\partial z^2} + (k_0^2 \epsilon - \beta^2) H_y = 0$$

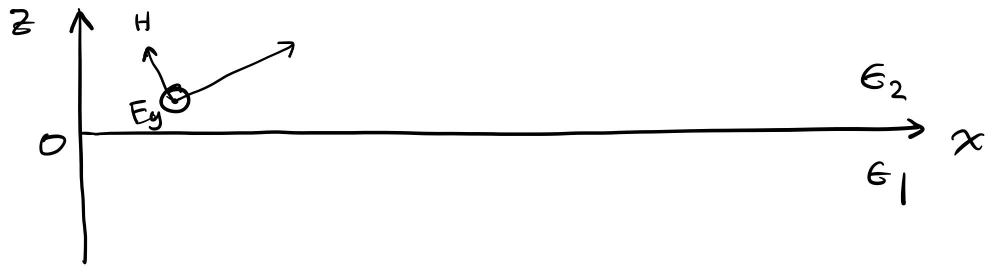
So
$$\begin{cases} k_1^2 = k_0^2 \epsilon_1 - \beta \\ k_2^2 = k_0^2 \epsilon_2 - \beta \end{cases} \qquad ②$$



Combing ① and ②

$$\beta = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}$$

Case II. TE wave



for $z > 0$

$$\begin{cases} E_y(z) = A_2 e^{i\beta x} e^{-k_2 z} \\ H_x(z) = -i A_2 \frac{1}{\omega\mu_0} k_2 e^{i\beta x} e^{-k_2 z} \\ H_z(z) = A_2 \frac{\beta}{\omega\mu_0} e^{i\beta x} e^{-k_2 z} \end{cases}$$

for $z < 0$

$$\begin{cases} E_y(z) = A_1 e^{i\beta x} e^{k_1 z} \\ H_x(z) = i A_1 \frac{1}{\omega\mu_0} k_1 e^{i\beta x} e^{k_1 z} \\ H_z(z) = A_1 \frac{\beta}{\omega\mu_0} e^{i\beta x} e^{k_1 z} \end{cases}$$

B.C. E_y and H_x are continous,

$$\Rightarrow \begin{cases} A_1 = A_2 \\ A_1(k_1 + k_2) = 0. \end{cases}$$

For a confined wave. $Re(k_1) > 0$, $Re(k_2) > 0$,

So to satisfy ③. $A_1 = 0$, $A_2 = 0$, $\Rightarrow$ $\boxed{\text{no wave exist!}}$

Conclusion:

SPP only exist for TM polarization!

Dispersion relation of SPP.

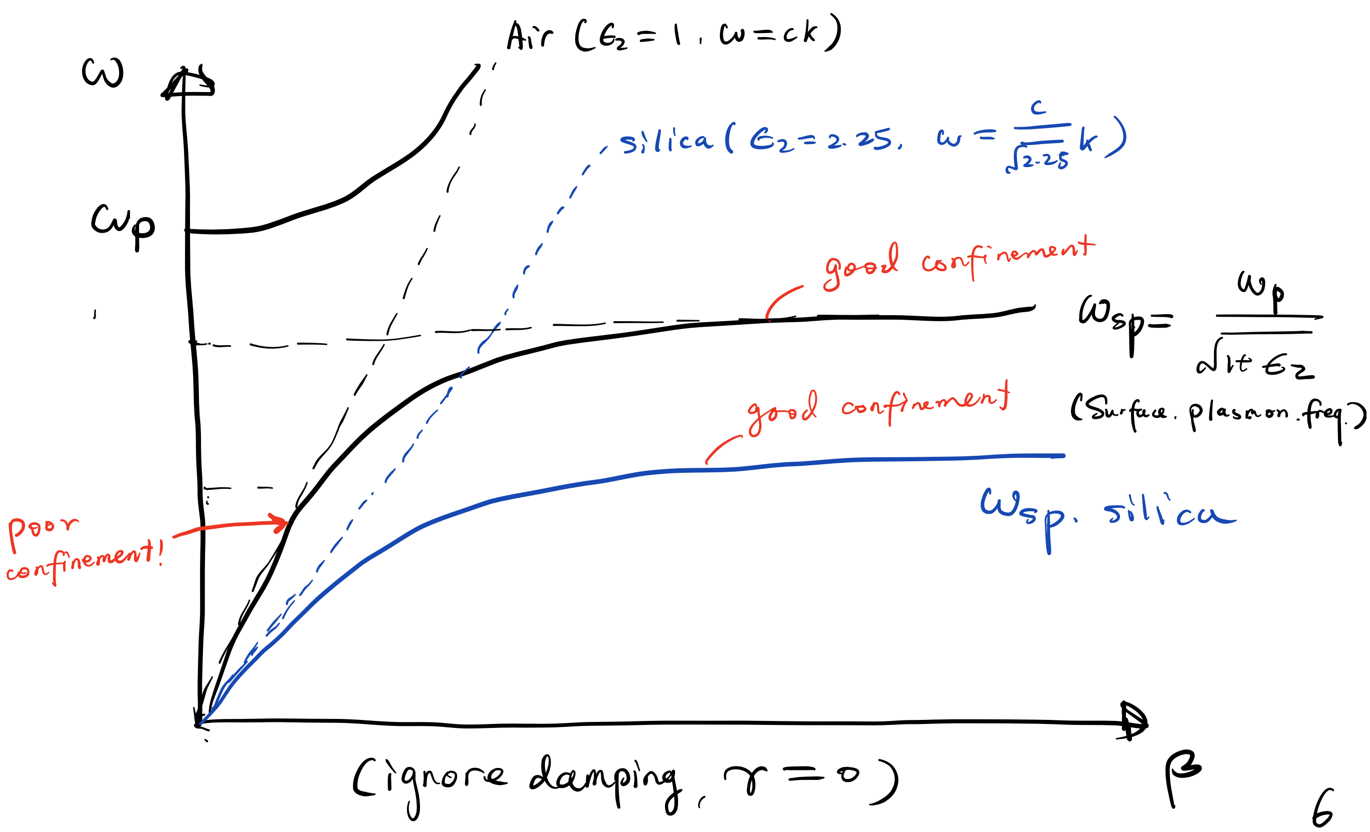
① Ignoring the damping.

$$\beta = k_0\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}} = \frac{\omega}{c}\cdot\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}$$

$$\Rightarrow \omega = c\cdot\beta\cdot\sqrt{\frac{\epsilon_1+\epsilon_2}{\epsilon_1\epsilon_2}} = c\cdot\beta\cdot\sqrt{\frac{\epsilon_1(\omega)+\epsilon_2}{\epsilon_1(\omega)\epsilon_2}} \quad \boxed{\epsilon_1(\omega) = 1-\frac{\omega_p^2}{\omega^2}}$$

Drude dielectric function

(ignore damping, $Im(\epsilon_1)=0$)



Three distinct frequency regions:

1). $0 < \omega < \frac{\omega_p}{\sqrt{1+\epsilon_2}}$

both ϵ_1 and $\epsilon_1 + \epsilon_2$ are negative, $\beta = \frac{\omega}{c}\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}$ is real. when ω is very small. $|\epsilon_1|$ is large (very negative) spp dispersion curve approaches the light line.

2) $\frac{\omega_p}{\sqrt{1+\epsilon_2}} < \omega < \omega_p$.

$\epsilon_1 < 0$, $\epsilon_1 + \epsilon_2 > 0$, β is imaginary, no propagating mode

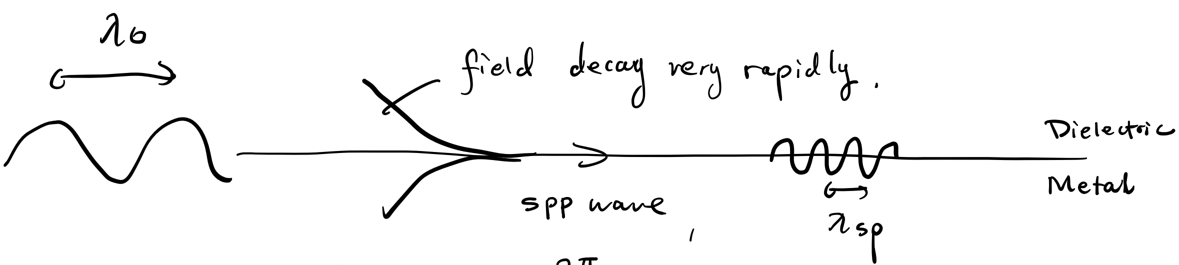
3) $\omega > \omega_p$. $\epsilon_1 > 0$, $\epsilon_2 > 0$ β is real again.

At very high freq. $\epsilon_m \rightarrow 1$, $\omega = c\beta\sqrt{\frac{1+\epsilon_2}{\epsilon_2}}$

Comments

1. When $\omega \sim \omega_{sp}$, β can be very large.

$$k_1 = \sqrt{\beta^2 - k_0^2 \epsilon_1}$$
$$k_2 = \sqrt{\beta^2 - k_0^2 \epsilon_2}$$ are very large. $\Rightarrow$ very good confinement!



also $\lambda_{sp} = \dfrac{2\pi}{\beta} << \lambda_0$

2. For typical metals (Ag, Au...) confinement is good is visibe and near-IR frequencies. In mid-IR, THz frequency, confinement vanishes.

> Reason? At low frequencies, field can hardly penetrate into metal and establish charge osillations

3. $\beta >> k_0$, momentum mismatch.

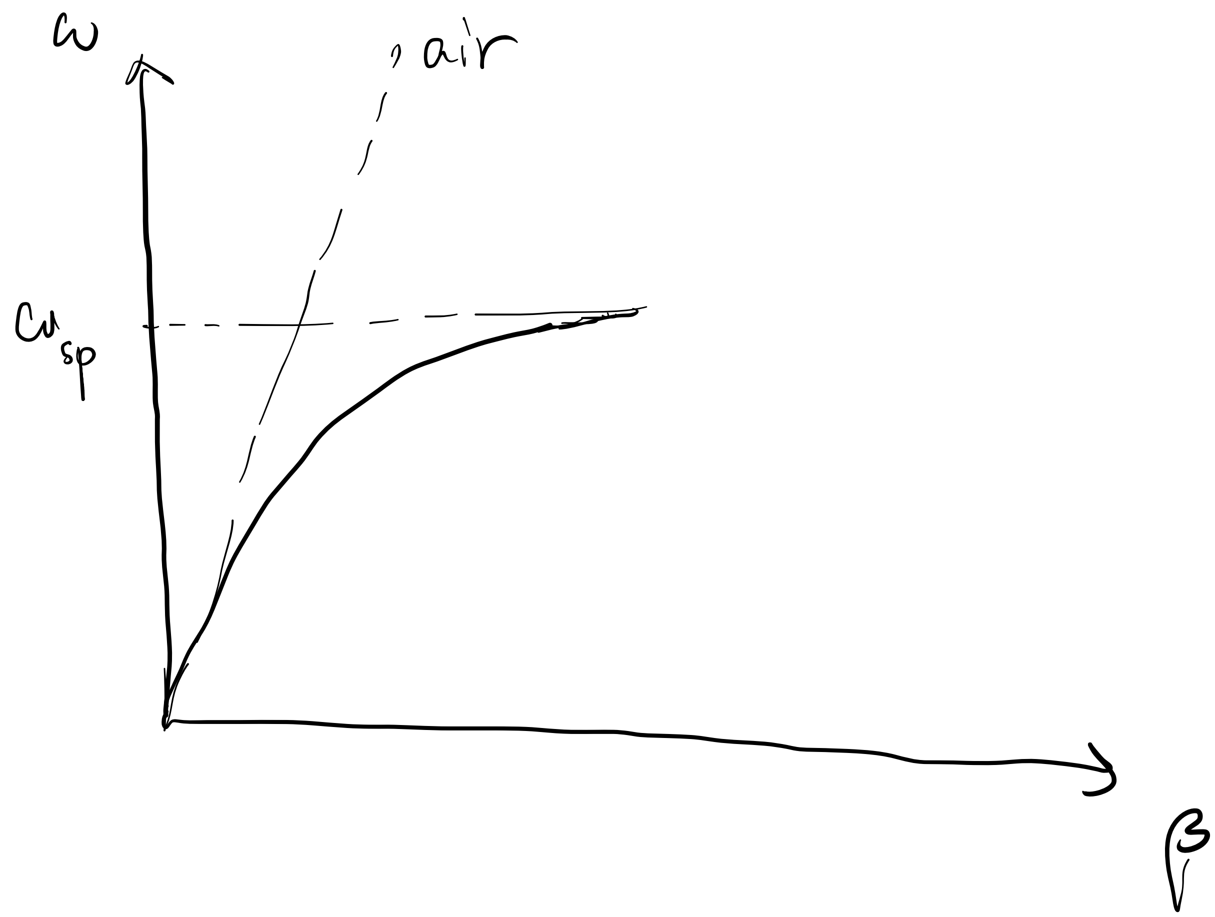
   require grating or prism to couple free-space light into spp.

4. At high β, $\omega \to \omega_{sp} = \frac{\omega_p}{\sqrt{1+\epsilon_2}}$

5. When igoring damping, β can go to infinity.

② Considering damping

$\epsilon_1(\omega)$ is complex, $\beta = \frac{\omega}{c}\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}$ is also complex.



Comments:

① When considering damping, β cannot go to infinity.
SPP approach a maximum, finite β at ω_{sp}

$\lambda_{sp} = \frac{2\pi}{Re(\beta)}$ can also reach a limit!

② propagation length of SPP $\simeq (2\,Im(\beta))^{-1}$.
For typical metals such as Au, Ag, $L_{spp} \sim 10 \sim 100\,\mu m$ in the visible regime.

③ when $\omega \to \omega_{sp}$, better confinement, higher damping.

eg. at silver/air interface.

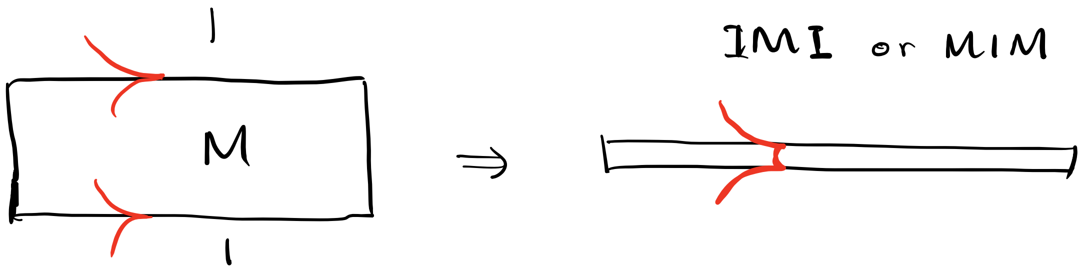
$\lambda_0 = 450$ nm, $L_{spp} \simeq 16\,\mu m$, decay length $\hat{z} = \left|\frac{1}{k_z}\right| = 180$ nm

$\lambda_0 = 1.5\,\mu m$, $L_{spp} \simeq 1080\,\mu m$. $\hat{z} \simeq 2.6\,\mu m$.

Trade-off between confinement and loss!

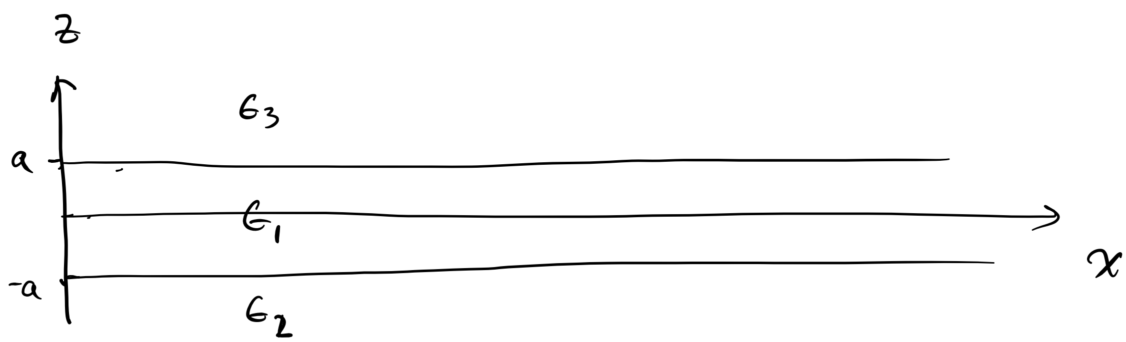
2. SPP in multi-layer systems

IMI or MIM



when t is comparable or smaller than the field decay length. SPP mode at top and bottom interface will be coupled!

$\Rightarrow$ $\boxed{\text{new dispersion!}}$



For TM mode:

when $z > a$

$$\begin{cases} H_y = A e^{i\beta x} e^{-k_3 z} \\ E_x = iA \dfrac{1}{\omega \epsilon_0 \epsilon_3} k_3 e^{i\beta x} e^{-k_3 z} \\ E_z = -A \dfrac{\beta}{\omega \epsilon_0 \epsilon_3} e^{i\beta x} e^{-k_3 z} \end{cases}$$

When $z < a$:

$$\begin{cases} H_y = B e^{i\beta x} e^{k_2 z} \\ E_x = -iB \dfrac{1}{\omega \epsilon_0 \epsilon_2} k_2 e^{i\beta x} e^{k_2 z} \\ E_z = -B \dfrac{\beta}{\omega \epsilon_0 \epsilon_2} e^{i\beta x} e^{k_2 z} \end{cases}$$

When $-a < z < a$ (core region), modes localized at the bottom and top interface are coupled:

$$\begin{cases} H_y = C e^{i\beta x} e^{k_1 z} + D e^{i\beta x} e^{-k_1 z} \\ E_x = -iC \dfrac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta x} e^{k_1 z} + iD \dfrac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta x} e^{-k_1 z} \\ E_z = C \dfrac{\beta}{\omega \epsilon_0 \epsilon_1} e^{i\beta x} e^{k_1 z} + D \dfrac{\beta}{\omega \epsilon_0 \epsilon_1} e^{i\beta x} e^{-k_1 z} \end{cases}$$

B.C. H_y and E_x are continous..

At $z = a$

$$\begin{cases} A e^{-k_3 a} = C e^{k_1 a} + D e^{-k_1 a} \\ \dfrac{A}{\epsilon_3} k_3 e^{-k_3 a} = -\dfrac{C}{\epsilon_1} k_1 e^{k_1 a} + \dfrac{D}{\epsilon_1} k_1 e^{-k_1 a} \end{cases}$$

At $z = -a$

$$\begin{cases} B e^{-k_2 a} = C e^{-k_1 a} + D e^{k_1 a} \\ -\dfrac{B}{\epsilon_2} k_2 e^{-k_2 a} = -\dfrac{C}{\epsilon_1} k_1 e^{-k_1 a} + \dfrac{D}{\epsilon_1} k_1 e^{k_1 a} \end{cases}$$

Also wave equation requires $k_i^2 = \beta^2 - k_0^2 \epsilon_i$

$i = 1.2.3$

$$\Rightarrow e^{-4k_1 a} = \frac{\frac{k_1}{\epsilon_1} + \frac{k_2}{\epsilon_2}}{\frac{k_1}{\epsilon_1} - \frac{k_2}{\epsilon_2}} \cdot \frac{\frac{k_1}{\epsilon_1} + \frac{k_3}{\epsilon_3}}{\frac{k_1}{\epsilon_1} - \frac{k_3}{\epsilon_3}} \qquad (1)$$

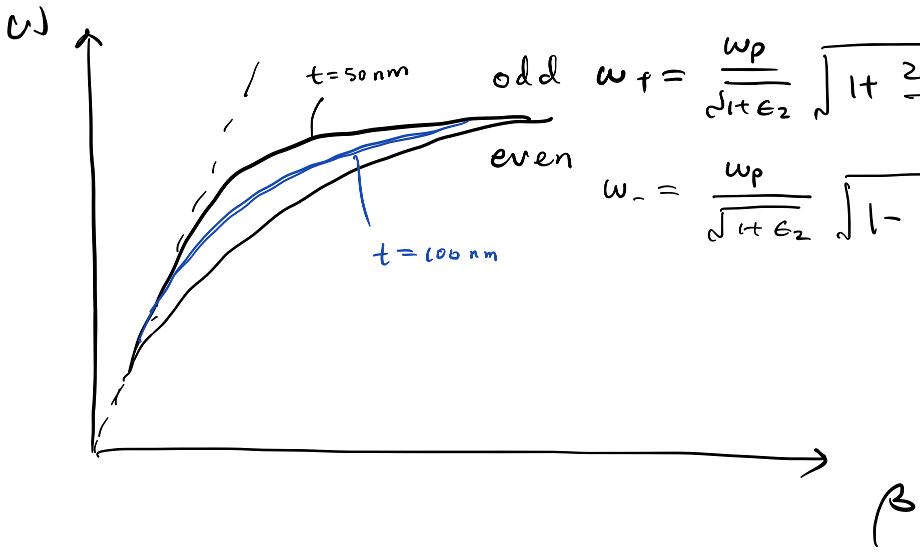
when $a \to \infty$, (1) becomes $\frac{k_2}{k_1} = -\frac{\epsilon_2}{\epsilon_1}$

Special case: $\epsilon_2 = \epsilon_3$, $k_2 = k_3$. (1) becomes

$\tanh k_1 a = -\frac{k_2 \epsilon_1}{k_1 \epsilon_2}$ odd mode ($E_x(z)$ is odd, $H_y(z)$ and $E_z(z)$ even)

$\tanh k_1 a = -\frac{k_1 \epsilon_2}{k_2 \epsilon_1}$ even mode ($E_x(z)$ is even, $H_y(z)$, $E_z(z)$ are odd)

(1) For IMI geometry



$$\omega_+ = \frac{\omega_p}{\sqrt{1+\epsilon_2}} \sqrt{1 + \frac{2\epsilon_2 e^{-2\beta a}}{1+\epsilon_2}}$$

$$\omega_- = \frac{\omega_p}{\sqrt{1+\epsilon_2}} \sqrt{1 - \frac{2\epsilon_2 e^{-2\beta a}}{1+\epsilon_2}}$$

Comments:

① $\omega_+ > \omega_{sp}$, $\omega_- < \omega_{sp}$.

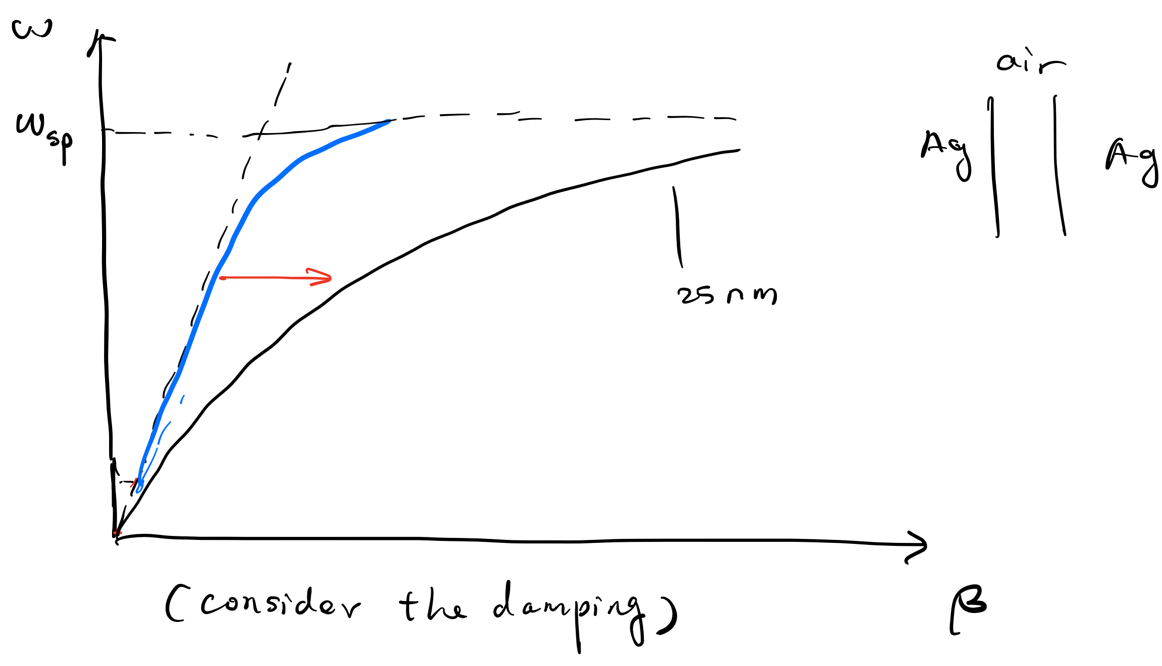
(ω_{sp} is for single interface)

② For same ω, when $t \downarrow$

confinement of odd mode $\downarrow$

" " even " $\uparrow$

② For MIM geometry



(consider the damping)

. MIM structure can improve the confinement

(even $\omega < \omega_{sp}$, β is large!)