Lecture 10: Chirp, self-phase modulation.

Learning objectives:

① Chirp.

② SPM

1. Chirp

last lecture:

Nonlinear Schrodinger Eq: $\frac{\partial A}{\partial z} + \frac{1}{2} i \beta_2 \frac{\partial^2 A}{\partial t^2} = i\gamma |A|^2 A$

Set nonlinearity $\gamma = 0$.

$$\Rightarrow \boxed{\frac{\partial A}{\partial z} = -\frac{1}{2} i \beta_2 \frac{\partial^2 A}{\partial T^2}}$$

pulse evolution with GVD only

Input Gaussian pulse: $A(0, T) = e^{-\frac{T^2}{2T_0^2}}$

Output pulse. $A(z, T) = \frac{T_0}{\sqrt{T_0 - i\beta_2 z}} e^{-T^2/2(T_0^2 - i\beta_2 z)}$ — complex.

$$= |A(z,t)| \cdot e^{i\phi(z,t)}$$

After propagation for a distance z, the pulse accumulate a phase, which is a function of both z and t.

$$\phi(z,T) = -\frac{sgn(\beta_2)(z/L_0)}{1+(z/L_0)^2}\frac{T^2}{T_0^2} + \frac{1}{2}\arctan\left(sgn(\beta_2)\cdot\frac{z}{L_0}\right)$$

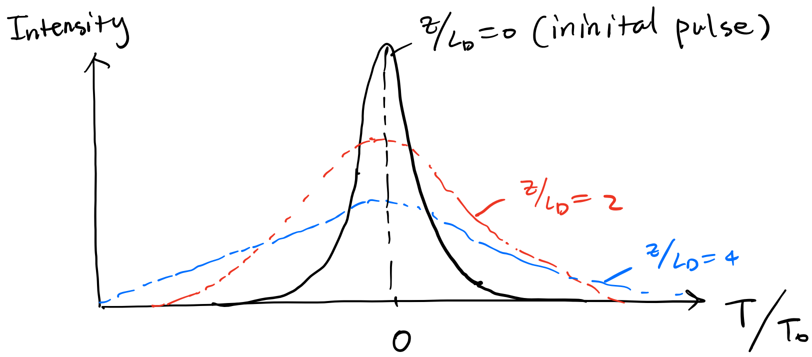
$$E(z,T) = A(z,T)\, e^{i(k_0 z - \omega_0 T)} = |A(z,t)|\, e^{ikz} \cdot e^{i\omega_0 t} \cdot e^{i\phi(z,t)}$$

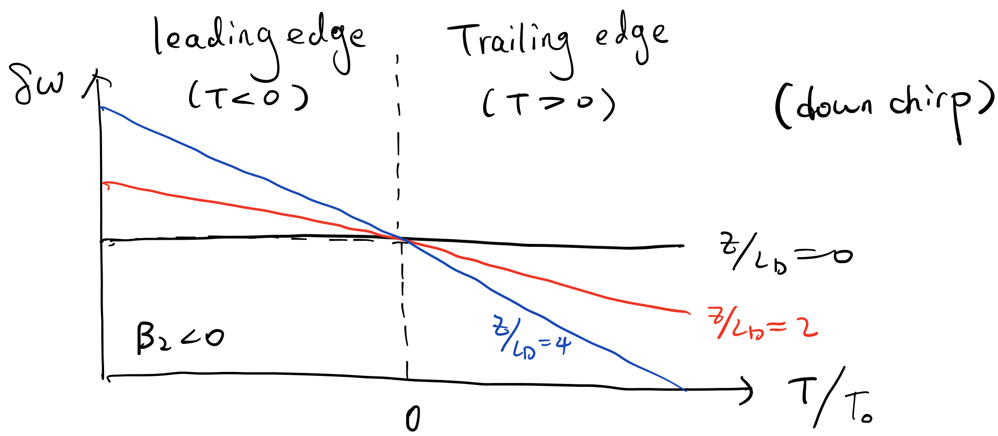
Meaning: the pulse has a frequency chirp around ω_0.

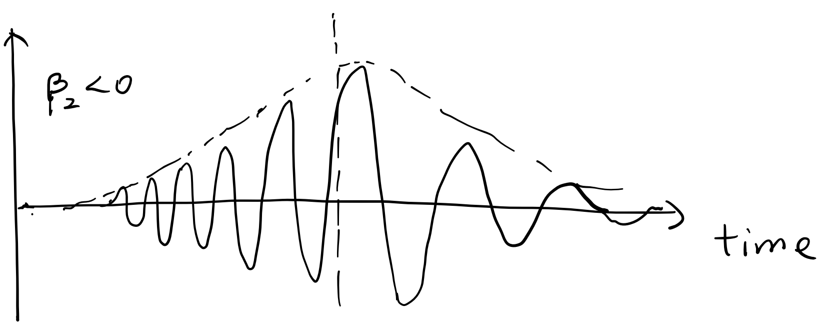
"Instantaneou frequency" $= \omega - \frac{\partial\phi}{\partial t} = \omega + \delta\omega$

$$\delta\omega = -\frac{\partial\phi}{\partial t} = \frac{sgn(\beta_2)(z/L_0)}{1+(z/L_0)^2} \cdot \frac{T}{T_0^2}$$

↑ chirp





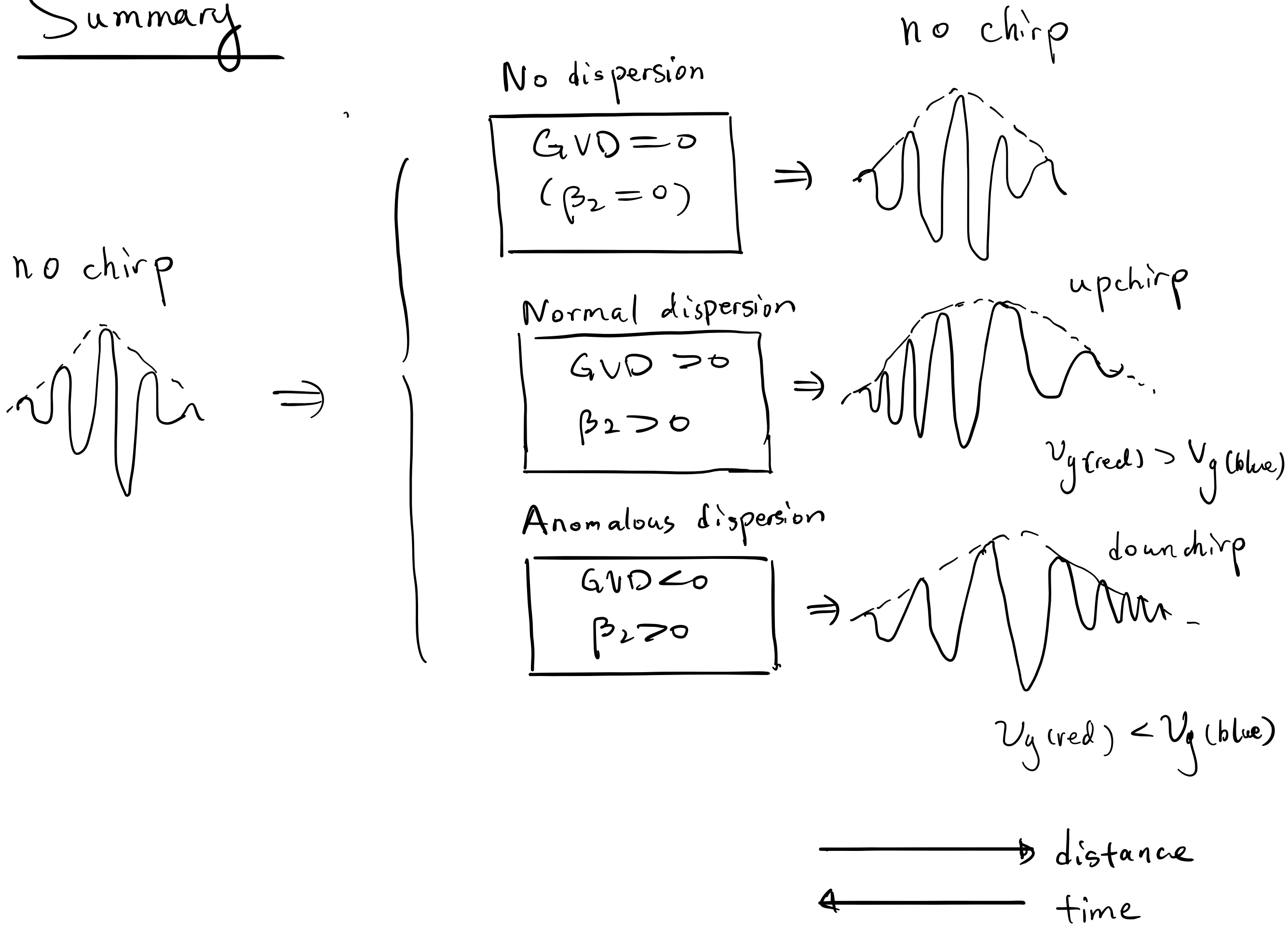


For $\beta_2 < 0$; $\frac{d}{d\omega}\left(\frac{1}{v_g}\right) < 0$.

$$\Rightarrow -\frac{1}{v_g^2}\frac{dv_g}{d\omega} < 0$$

$$\Rightarrow \frac{dv_g}{d\omega} > 0.$$ (higher freq components travels faster, $V_g(\text{red}) < V_g(\text{blue})$)

Summary



Note: With linear dispersion, pulse gets broadened in time domain, but the spectrum remains the same. (no new freq. generation!)

When start with chirped Gassian pulse:

Suppose input pulse has initial amplitude:

$$A(0,T) = \exp\left[-\frac{(1+iC)}{2}\frac{T^2}{T_0^2}\right]$$

where C is chirp parameter, can be determined by measuring spectral width

$C>0$. up chirp

$C<0$. down chirp.

$$\tilde{A}(0,\omega) = \left(\frac{2\pi T_0^2}{1+iC}\right)^{1/2} \cdot \exp\left[-\frac{\omega^2 T_0^2}{2(1+iC)}\right]$$

$1/e$ spectral width: $\Delta\omega = \dfrac{\sqrt{1+C^2}}{T_0}$

without chirp. $(C=0)$. $\Delta\omega \cdot T_0 = 1$

with chirp $(C \neq 0)$ $\Delta\omega \cdot T_0 = \sqrt{1+C^2}$. enlarged by $\sqrt{1+C^2}$!

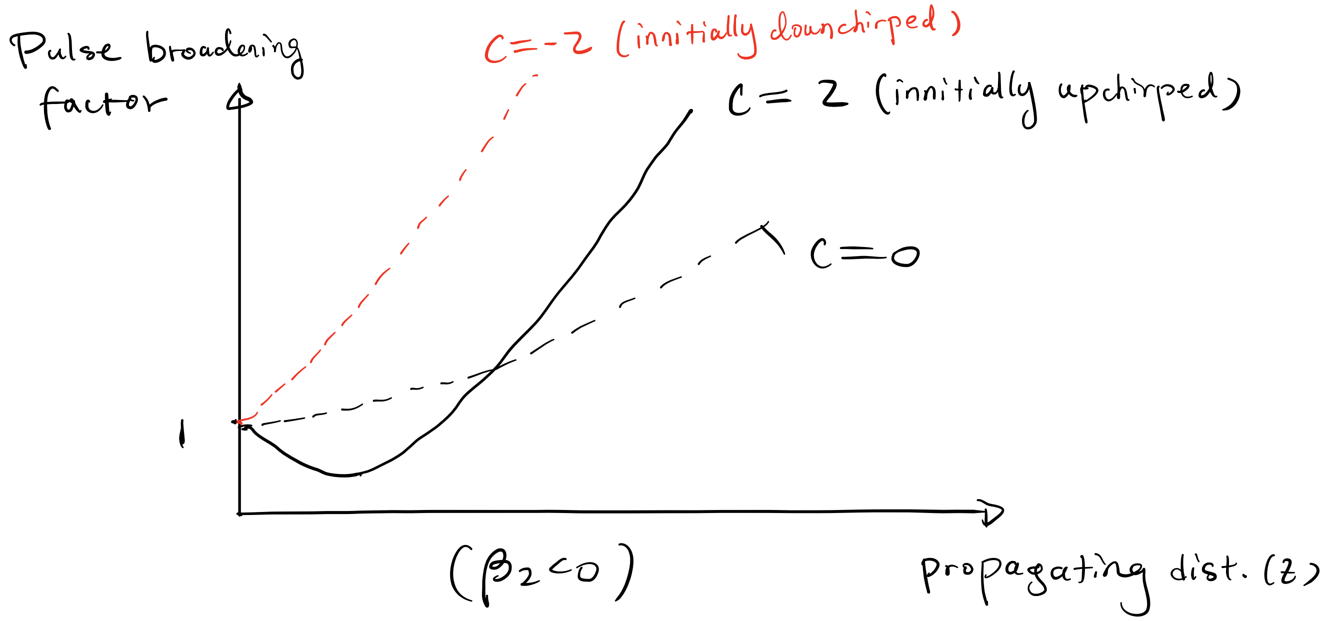
After propagation for a distance z:

$$\tilde{A}(z,T) = \frac{T_0}{[T_0^2 - i\beta_2 z(1+iC)]^{1/2}} \cdot \exp\left(-\frac{(1+iC)T^2}{2[T_0^2 - i\beta_2 z(1+iC)]}\right)$$

$$T_1 = T_0\sqrt{\left(1+\frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0}\right)^2}$$

And the chirp parameters changes to

$$C_1(z) = C + (1+C^2)\left(\frac{\beta_2 z}{T_0^2}\right)$$



Comments:

① Initially chirped pulses may get broadened or compressed depending on $\beta_2 \cdot C$

② When $\beta_2 \cdot C > 0$, broadened monotonically

③ When $\beta_2 C < 0$, compressed first, then gets broadened.

Dispersion management

- Use consecutive fiber with opposite-sign β_2

$$\beta_2^A L_A = -\beta_2^B L_B$$

- Can be advantageous to have dispersion. — inhibits nonlinear effects.

2. Self-phase modulation (SPM)

NLS, drop dispersion, only consider Kerr nonlinearity

i.e. $$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + \cancel{\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2}} + i\gamma|A|^2 A. \quad ①$$

normalized amplitude

Solution: $A(z,T) = \sqrt{P_0}\, e^{-\frac{\alpha}{2}z}\, U(z,T)$, plug in ①

$$\frac{\partial U}{\partial z} = \frac{i e^{-\alpha z}}{L_{NL}} |U|^2 U$$

$$L_{NL} \rightarrow \frac{1}{\gamma P_0}$$

Solution: $U(L,T) = U(0,T)\cdot e^{i\phi_{NL}(L,T)}$

$$\phi_{NL} = |U(0,T)|^2 \frac{L_{eff}}{L_{NL}}, \quad L_{eff} = \frac{1-e^{-\alpha L}}{\alpha} = \begin{cases} L, & \alpha L \ll 1 \\ \frac{1}{\alpha}, & \alpha L \gg 1 \end{cases}$$

$\Rightarrow$ Intensity-dependent phase shift!

If $|U(0,0)|=1$, $\phi_{max} = \left.\frac{L_{eff}}{L_{NL}}\right|_{\alpha=0} = \frac{L}{L_{NL}}$

$\Rightarrow \phi_{max} = 1$ at $L = L_{NL}$

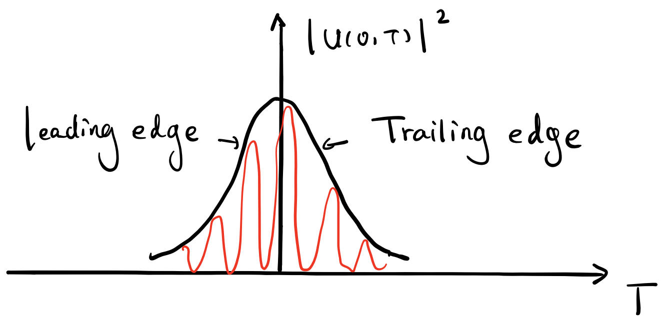
e.g. For SiO_2 fiber, $\gamma = 2\ W^{-1}km^{-1}$. $L_{NL} = 50\ km$ for $P_0 = 10\ mW$

$L_{NL} = 100\ m$ for $P_0 = 5W$.

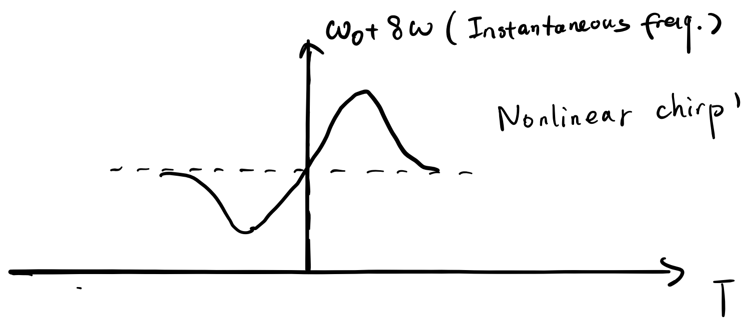
Frequency chirp:

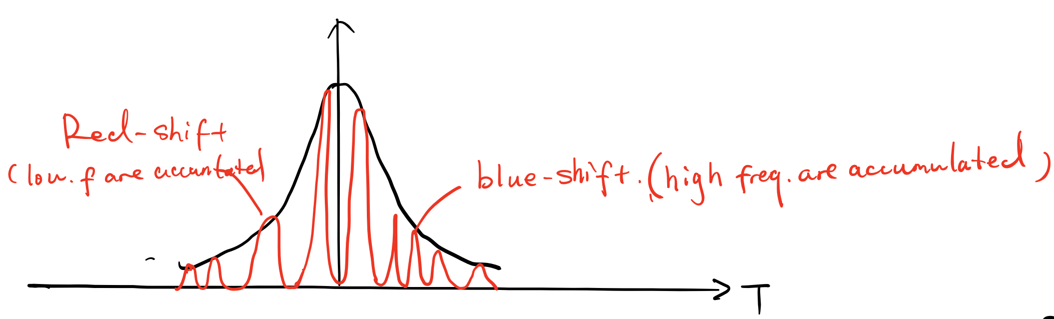
$$\delta\omega(T) = -\frac{\partial \phi_{NL}}{\partial T} = -\left(\frac{L_{eff}}{L_{NL}}\right)\cdot\frac{\partial}{\partial T}\left[|U(0,T)|^2\right]$$

$z=0$:



$z=z$

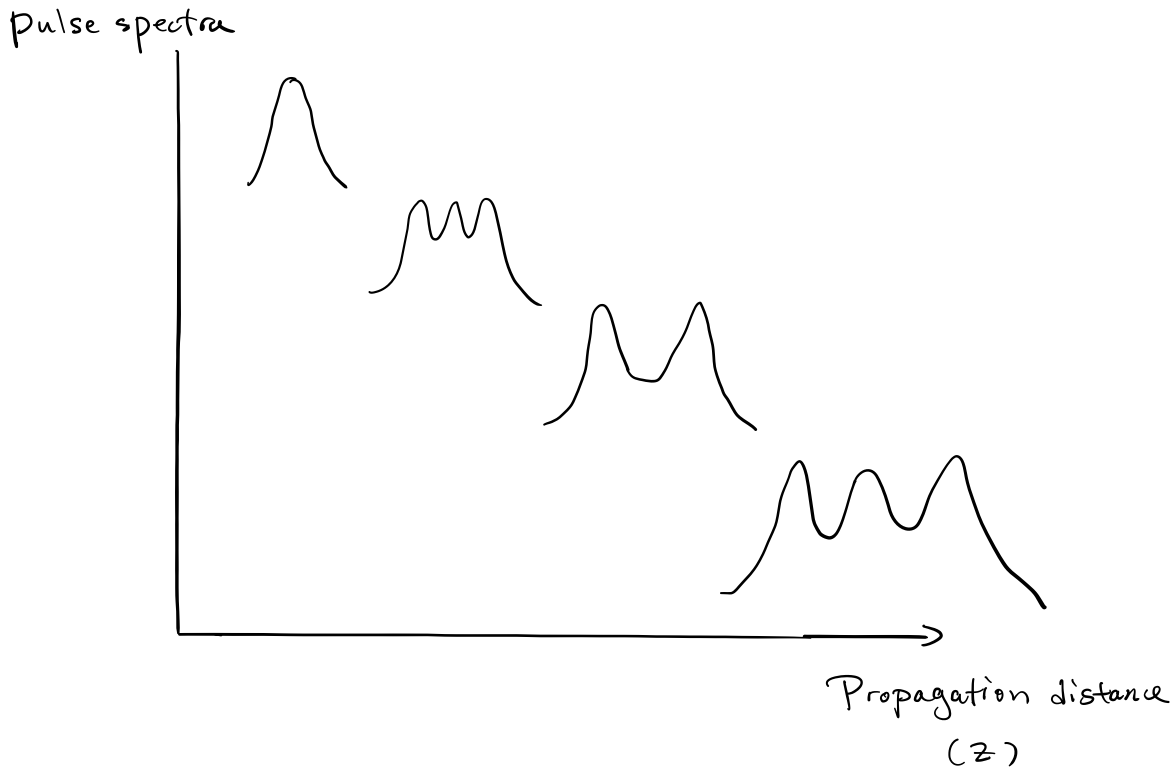




Comments:

1. SPM doesn't change pulse envelope, but changes spectrum, more new freq. are generated.

2. Result: spectrum broadening!



3. In case of unchirped input pulses, SPM always leads to spectral broadening. With linearly chirped input pulse, spectral narrowing can occur!

4. Chirp / spectral broadening factor depends on pulse shape!

$$\delta\omega(T) = \frac{2m}{T_0} \frac{L_{eff}}{L_{NL}} \left(\frac{T}{T_0}\right)^{2m-1} \cdot \exp\left[-\left(\frac{T}{T_0}\right)^{2m}\right]$$

$m=1$, Gassian pulse

$m \uparrow$, steeper leading and trailing edges.